DISCRETE MATHEMATICAL STRUCTURES (BMA-204)

Teacher Name:

Prof. (Dr.) Ramautar

Course Structure

Sr. No.	Course Code	Course Name	Credits	Details of Sessional Marks				ESM	Total Marks
				CT	TA	Lab	Total		
4	BMA-204	Discrete Mathematical Structures	4 (3-1-0)	30	20	-	50	50	100

Prerequisite:

Course Content:

Unit-1: Fundamentals of Logic

Propositional Logic: Propositions, Basic logic operations and truth tables, Tautologies, Contradictions, Contigency, Algebra of propositions, Logical equivalence: the laws of logic, Logical implication: Rules of inference, Logical analysis of argument, Some computing application (Normal forms), Functionally complete set of operations, Formal proofs.

First Order Predicate Logic: Predicates & quantifiers, Nested quantifiers, Use of quantifiers, Rules of inference, Validity of arguments and proof methods.

Unit-2: Set Theory, Relations and Functions

Set Theory: Sets & subsets, Venn diagrams, set operations and laws, countable set, Cartesian product, Cardinality, Principle of inclusion- exclusion.

Relations: Relation, Representation & properties, n-ray relations and applications, Composition of relations, Closures of relations, Equivalence relation & partitions, partial orders, compatibility relation.

Functions: Functions and its types, Inverse function, Composition of functions, Special functions, Recursively defined functions, Computational Complexity, Analysis of algorithms.

Theorem Proving Techniques: Mathematical induction (weak, strong, structural) and its applications, Proof by contradiction, Pigeonhole principle.

Unit-3: Algebraic Structures and Coding Theory

Algebraic Structures: Definition, Properties, Semi group, Monoid, Group, properties of groups, Subgroup, Cyclic group, Cosets and Lagrange’s theorem, Permutation groups, Normal subgroup, Homomorphism and isomorphism of groups, Congruence relation, Rings and Fields. Example and standard results.

Coding Theory: Elements of coding theory, Hamming matric, Parity-check and generator matrices, Coding and error detection, Group codes: decoding with coset leaders and error correction, Hamming matrices.

Unit-4: Partially Ordered Structures

Posets: Definitions, ordered set, Hasse diagram, isomorphic ordered set, well ordered set, Minimal and Maximal elements, LUB &GLB etc.

Lattices: Definition & Properties, Product Lattices, Isomorphic Lattices, Applications, Types of Lattices.

Boolean Algebra: Definitions & Properties, SOP & POS forms, Logic gates and minimization of circuits, Karnaugh maps, Quine-McClusky method.

Trees: Definition & Examples and Properties, Rooted tree, Binary tree, Tree traversal, application in computer science and engineering.

Unit-5: Combinatorics and Graph Theory

Combinatorics: Discrete numeric functions and properties, Recurrence relations and their applications (modeling), various methods of solutions, system of recurrence relations, OGF & EGF, properties, applications: solution of recurrence relations and combinatorial problems.

Graphs: Graphs and graph models, terminology, matrices associated with graphs, Isomorphism, Special types of graphs, connectedness, Euler and Hamilton graphs with their applications, trees with properties, MST, planer graphs and applications, criteria of planarity, Graph coloring and coloring models, directed graphs.

Text and Reference Books:

1. Trembley, J.P. & R. Manohar, "Discrete Mathematical Structures with applications to Computer Science", McGraw Hill.
2. Kenneth H. Rosen, "Discrete Mathematics and its Applications", McGraw Hill.
3. Ralph, P. Garimaldi, "Discrete& Combinatorial Mathematics" Pearson Publication, Asia.
4. Deo, narsingh, "Graph Theory with applications to Engineering & Computer Science", PHI.
5. Krishnamurthy, V., "Combinatorics Theory & Application", East-West Press Pvt. Ltd., New Delhi.

Course Outcomes:

1. Understand concepts of Logic and various inference mechanisms using logic. (Understand)
2. Understand Set theory, functions, relations and the concepts of theorem proving. (Understand)
3. Explain algebraic structure and coding theory. (Understand)
4. Understand and apply concepts of partially ordered structures, Boolean algebra and trees in various application of computer science domain. (Understand, Apply)
5. Understand and apply graph theory and concepts of recurrence relation in system modeling. (Understand, Apply)

ASSIGNMENT-1

NOTE: The completed assignement must be submitted before 30/03/2019.

1.(a) (i) Name two persons who contributed to the development of logic.
(ii) Express in predicate logic :
"Some people tell the truth some of the time, others never lie".
(iii) Write a parse tree of : $\rightarrow \wedge P \rightarrow QR \rightarrow \wedge P \urcorner Q \vee \urcorner Q \urcorner R$.
(iv) Use Venn diagram to examine the validity of : $A \rightarrow B$, $B \rightarrow C \Rightarrow A \rightarrow C$.
(v) Use the following in predicate logic and by Venn diagram :
"No beggar is a thief".

(b) Write a program in C/C++ to examine whether the following system specifications are consistent :
"Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save files, then the system software is not being upgraded ".

(c) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\urcorner Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \urcorner S(x))$, $and\ \exists x \urcorner P(x)$ are true , then $\exists \urcorner R(x)$ is true.

(d) Use a proof by contraposition and a proof by contradiction to examine the statement :
"If n is an integer and $3n + 2$ is even , then n is even".

2.(a) (i) Use quantifiers to define : $A \subset B$.
(ii) Use quantifiers to define : f is one-to-one.
(iii) Express mathematical induction as a rule of inference.
(iv) Who introduced $big - \Omega\ and\ big - \Theta$ notations to describe the growth of functions.

(b) Examine the following for a countable set :
(i) the set of all positive integers.
(ii) the set of all C/C++ programs.

(c) Develop a program in C/C++ to compute an immediate predecessor relation matrix from the partial order relation matrix :

$$M_{\leq} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(d) State the principle of inclusion-exclusion and its applications.
How many ways are there to assign five different jobs to four different employees if every employee is assigned atleast one job?

(e) State some elegant applications of the pigeon-hole principle.
How many numbers must be selected from the set $\{1, 3, 5, , 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

3.(a) (i) State some real-life problems where the notion of partial order/total order arises ?

(ii) Name some applications of trees as a partially ordered set.

(iii) Name some uses of the lattices.

(iv) How are Boolean algebras useful for the Computer Scientists and Electronic Engineers ?

(b) Develop a partially ordered structure on the set of all positive even factors of 308 and examine it for a totally ordered set , well ordered set , lattice with its types and Boolean algebra.

(c) State the purpose of minimization of the Boolean functions and two methods for the purpose. Use any one to simplify the SOP expansion:

$$wxy\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}xyz + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}z$$

4.(a) (i) Examine $K_{2,2,3}$ for a planar graph.

(ii) Name important applications of graph coloring.

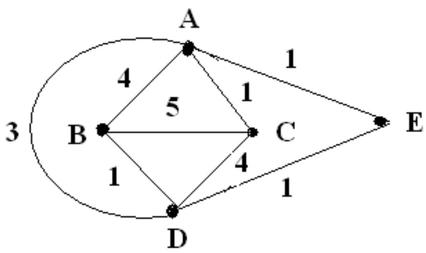
(iii) What is discrete numeric function described by EGF :e^{z^2}.

(iv) Name two applications of Polya's enumeration theorem.

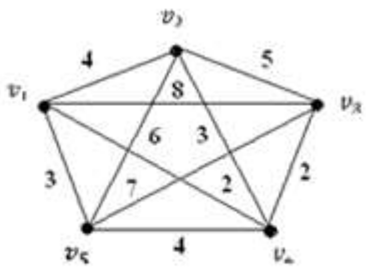
(b) A ship carries 48 flags, 12 each of the colors red, white, blue and black. 12 of these flags are placed on a vertical pole in order to communicate a signal to other ship. Determine the number of signals which use an even number of blue flags and an odd number of black flags.

(c) There are two kinds of particles inside a nuclear reactor. In every second, an α particle will split into three α particles and one β particle, and a β particle will split into two α particles and one β particle. If there is a single α particle in a reactor initially, Develop a recurrence- relation model for the number of α and β particles in the reactor at time n.

(d) Solve the Chinese postman problem for the weighted graph :



(e) Use Prim's algorithm (matrix version) to find MST for the weighted graph :



ASSIGNMENT-2

DISCRETE MATHEMATICAL STRUCTURES (BMA-204)

II CSE/IT

NOTE: The completed assignement must be submitted before 05/04/2019.

1. (a) State four applications of logic in Computer Science.

(b) Write the following statement as an implication in two ways :

None but the brave deserve the fair.

(c) Use quantifiers to define the following :

(i) f is one-to-one (ii) $A \subset B$

(d) How can the efficiency of an algorithm be analyzed ?

(e) State two applications of cosets and Lagrange's theorem.

(f) State the importance of a group code.

(g) State four applications of binary trees.

(h) State applications of n-ary relations and lattices.

(i) If $G(z)$ is the generating function for the sequence $\{a_n\}$, find the generating function for the sequence: $0,\ a_0, \frac{a_1}{2}, \frac{a_2}{3}, \frac{a_3}{4}, ...$

(j) State two special types of graphs to model Local Area Networks.

2. (a) Develop a program in C/C++ to examine :

(i) $\neg(P \leftrightarrow Q)$ *and* $\neg P \leftrightarrow Q$ for logical equivalence.

(ii) $\left[(P \to Q) \wedge (Q \to R)\right] \to (P \to R)$ for a tautology.

(b) Examine the validity of the following arguments :

(i) There is a man whom all men despise. Therefore, atleast one man despises himself.

(ii) All horses are animals. Therefore, the head of a horse is the head of an animal.

(c) Establish the validity of the following argument or give a counter example to show that it is invalid:

$$\begin{array}{c} P \leftrightarrow Q \\ Q \longrightarrow R \\ R \vee \neg S \\ \neg S \longrightarrow Q \\ \hline \therefore S \end{array}$$

3. (a) Develop a program in C/C++ for the recognition of a partial order relation on a finite set.

(b) Examine the set $Z^+ \times Z^+ \times Z^+$ for a countable set.

(c) Illlustrate the application of mathematical induction to the analysis of computer algorithms with atleat one example.

(d) State some practical applications of functions in computer science. Examine the function for a bijection :

$f : R \times R \to R \times R$ defined by $f(x, y) = (x + y, 2x - y)$

If yes, find its inverse.

4. (a) State some important applications of the partially ordered structures.
Examine whether the set of positive divisors of 385 with divisibility relation yields a poset, totally ordered set, well-ordered set, a lattice and its types, and Boolean algebra.

(b) Use a Karnaugh map or Quine-McClusky method to find a minimal-sum-of products representation for

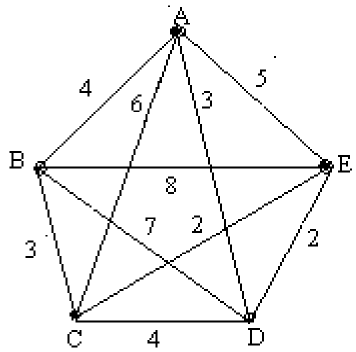
$$f(v, w, x, y, z) = \Pi M(1,2,4,6,9,10,11,14,17,18,19,20,22,25,26,27,30)$$

5. (a) There are two kinds of particles inside a nuclear reactor. In every second, an α particle will split into three α particles and one β particle, and a β particle will split into two α particles and one β particle. If there is a single α particle in a reactor initially,

(i) Develop a recurrence- relation model for the number of α and β particles in the reactor at time n.

(ii) Use generating functions to solve the recurrence relation model.

(b) (i) State two applications of Hamilton circuits.
Solve the travelling salesman problem for the weighted graph:



(ii) Illustrate how Euler's formula for planer graphs can be used to show that a simple graph is nonplaner.

6. (a) Prove that if n is an integer and 3n+2 is even, then n is even, using a proof by contraposition and a proof by contradiction.

(b) Let ABCD be a square with AB = 1. Show that if we select 5 points in the interior of this square, these are atleast two whose distance apart is less than $\frac{1}{\sqrt{2}}$.

(c) Use generating functions to find the number of ways in which a postage of 137 cents can be pasted on an envelop using 3-cent, 4-cent, and 20-cent stamps. Assume that the order the stamps are pasted on does not matter.

(d) Explain how trees can be used to model the computer file system and parallel processors.

ASSIGNMENT-3

DISCRETE MATHEMATICAL STRUCTURES (BMA-204)

II CSE/IT

NOTE: The completed assignement must be submitted before 08/04/2019.

1) **(a) State the negations of the statements:** $\forall x(x^2 > 2)$ **and** $\exists x(x^2 = 2)$.

   (b) Examine $(p \wedge q) \rightarrow (p \vee q)$ **for a tautology without using truth table.**

   (c) Using quantifiers, define one-to-one function and onto function.

   (d) State important applications of n-ary relations.

   (e) State two applications of algebraic structures.

   (f) State the importance of parity-check matrix.

   (g) State applications of posets and lattices.

   (h) State two applications of binary trees.

   (i) Find a generating function for the sequence defined by $a_r = 1+2+3+...+r$.

   (j) Examine Petersen graph for Hamilton graph and planar graph.

2) **(a) Find the principal disjunctive normal form and principal conjunctive normal form of the statement formula:**

$$(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$$

   (b) Establish the validity of the following arguments:

   i) **"If you send me an e-mail message, then I will finish writing the program. If you do not send me an e-mail message, then I will go to sleep early. If I go to sleep early, then I will wake up feeling refreshed. Therefore, if I do not finish writing the program, then I will wake up feeling refreshed."**

   ii) $(p \wedge t) \rightarrow (r \vee s)$

   $q \rightarrow (u \wedge t)$

   $u \rightarrow p$

   $\neg s$

   $\therefore q \rightarrow r$

   (c) Examine the validity of the following argument:

"All radioactive substances either have a very short life or have medical value. No uranium isotope that is radioactive has very short life. Therefore, if all uranium isotopes are radioactive, then all uranium isotopes have medical value."

3) (a) State the principle of inclusion-exclusion and its applications.
How many ways are there to assign five different jobs to four different employees if every employee is assigned at least one job?

(b) Use strong induction to prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

(c) Use a proof by contradiction to prove the pigeonhole principle.

14 students in a class appear at a university examination. Prove that there exist at least two among them whose seat numbers differ by a multiple of 13.

(d) Given A as below

A = {(-4,-20),(-3,-9),(-2,-4),(-1,-11),(-1,-3),(1,2),(1,5),(2,10),(2,14),(3,6),(4,8),(4,12)}

Define the relation R on A by (a,b)R(c,d) if ad = bc

Examine R for equivalence relation on A. If yes find Equivalence class [(4,8)].

4) (a) A binary relation '$\le$' on D_{30} is defined by **[10]**

$a \le b$ **if a divides b**

i) **Examine $\le$ for partial order relation.**
If yes,

ii) **Draw its Hasse diagram,**
iii) **Examine the poset for Lattice,**
iv) **Examine the same for a Boolean algebra.**

(b) Find a minimal-sum-of-products representation for the function:

$$f(x, y, z, w) = \Sigma m(0,1,2,8,10,11,14,15)$$

Using i) Karnaugh map, ii) Quine-McClusky method.

5) (a) A computer system consider a string of decimal digits a valid codeword if it contains a even number of 0 digits. Develop a recurrence relation model for the number of valid n-digit codewords.

(b) Use generating function to solve the recurrence relation

$$a_n = na_{n-1} + (-1)^n,\ n \ge 1$$

$$a_0 = 1$$

(c) Illustrate how the graphs can be used in modeling of the local area computer networks and Interconnection Networks for parallel computation.

(d) Explain how graph coloring/multicoloring can be used in a variety of different models.

6) (a) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

(b) A ship carries 48 flags, 12 each of the colors red, white, blue and black. 12 of these flags are placed on a vertical pole in order to communicate a signal to other ship. How many of these signals use an even number of blue flags and an odd number of black flags?

(c) Use Prim's algorithm (matrix version) find a minimal spanning tree for the graph: