

Subject Code: NCE 202

Subject Name: Hydraulics & Hydraulic Machines

DR. DEEPESH SINGH

DEPARTMENT OF CIVIL ENGINEERING

Harcourt Butler Technical University, Kanpur, India- 208002



UNIT - 1



DR. DEEPESH SINGH

DEPARTMENT OF CIVIL ENGINEERING

Harcourt Butler Technical University, Kanpur, India- 208002



MAIN FEATURES

✤DEPTH, WATER AREA, VELOCITY AND DISCHARGE ARE CONSTANT

ENERGY LINE, CHANNEL BOTTOM AND WATER SURFACE ALL ARE PARALLEL





MAIN FEATURES

✤DEPTH, WATER AREA, VELOCITY AND DISCHARGE ARE CONSTANT

ENERGY LINE, CHANNEL BOTTOM AND WATER SURFACE ALL ARE PARALLEL







Fig. 3.1 Definition sketch of uniform flow

Momentum Equation : $P_1 + W \sin \theta - F_1 - P_2 = M_2 - M_1$ 3.1



Since the flow is uniform,

$$P_1 = P_2$$
 and $M_1 = M_2$
 $W = \gamma AL$ and $F_\ell = \tau_0 PL$

where τ_0 = average shear stress on the wetted perimeter of length *P* and γ = unit weight of water. Replacing sin θ by S_0 (= bottom slope), Eq. 3.1 can be written as

Driving Force = Resisting Force

$$\gamma ALS_0 = \tau_0 PL$$
 Or $\tau_0 = \gamma \frac{A}{P} S_0 = \gamma RS_0$ 3.2

Expressing the average shear stress τ_0 as $\tau_0 = k\rho V^2$,

where k = a coefficient which depends on the nature of the surface and flow parameters, Eq. 3.2 is written as

$$k\rho V^2 = \gamma RS_0$$
 $V = C \sqrt{RS_0}$ Chezy formula Year 1769

where $C = \sqrt{\frac{\gamma}{\rho} \frac{1}{k}} =$ a coefficient which depends on the nature of the surface and the dimensions of *C* are $[L^{1/2} T^{-1}]$



For pipe flow, the Darcy–Weisbach equation is $h_f = f \frac{L}{D} \frac{V^2}{2g}$

where h_f = head loss due to friction in a pipe of diameter D and length L; f = Darcy-Weisbach friction factor. f is found to be a function of the Reynolds number $\left(\operatorname{Re} = \frac{VD}{v} \right)$ only.

For rough turbulent flows, *f* is a function of the relative roughness (ε_s/D) and type of roughness and is independent of the Reynolds number.

For rough boundaries and $Re > 10^5$

$$\frac{1}{\sqrt{f}} = -2\log\frac{\varepsilon_s}{D} + 1.14$$

(Karman-Prandtl equation)

DARCY–WEISBACH FRICTION FACTOR f

where $\varepsilon_s =$ equivalent sand grain roughness,

pipe of diameter D



for an open channel flow Darcy-Weisbach equation is

$$h_f = f \frac{L}{4R} \frac{V^2}{2g}$$

Slope = $S_f = S_0 = h_f / L = f V^2 / 8Rg$

which on rearranging gives
$$V = \sqrt{\frac{8g}{f}} \sqrt{R} \cdot \sqrt{h_f / L}$$

Same as Chezy's Equation $V = C \sqrt{RS_0}$ & $C = \sqrt{8 g / f}$



Fig. 3.2 Variation of f in fully rough flow



 $f \propto (k_s / R)^{1/3}$ $C = \sqrt{8 \text{ g/} f}$ $C \propto (R / k_s)^{1/6}$ $V = C \sqrt{RS_0}$ In Chezy's Equation Put C $V = \frac{1}{n} R^{2/3} S_0^{1/2}$ Manning's Equation $n \propto (k_s)^{1/6}$

where n = a roughness coefficient known as Manning's *n*.





For natural streams

Where d_{50} is in metres and represents the particle size in which 50 per cent of the bed



For coarse grained soil

Where d_{90} = size in metres in which 90 per cent of the particles are finer than d_{90} .





shutterstock.com - 1516656128





Image source: http://people.uwplatt.edu





Fig. 3.5(a) Typical velocity distribution in a narrow channel, $B/y_0 = 1.0$. (Ref. 4)



Dimension of n is $[L^{-1/3} T]$.

In Manning's Formula there is a term 1/n

1 has the dimension m $^{1/3}$ / s in MKS

In FPS system1 has the dimension (3.28)ft^{1/3} / s

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad \text{In FPS}$$

So *n* is always constant whichever system of units is used. Only dimension of *1* changes



Manning's n Roughness Coefficient

Table 3.2 Values of Roughness Coefficient n

SI. No.	Surface Characteristics	Range of n
(a)	Lined channels with straight alignment	
1	Concerete (a) formed, no finish	0.013-0.017
	(b) Trowel finish	0.011-0.015
	(c) Float finish	0.013-0.01
	(d) Gunite, good section	0.016-0.019
	(e) Gunite, wavy section	0.018-0.022
2	Concrete bottom, float finish, sides as indicated	
	(a) Dressed stone in mortar	0.015-0.01
	(b) Random stone in mortar	0.017-0.02
	(c) Cement rubble masonry	0.020-0.02
	(d) Cement-rubble masonry, plastered	0.016-0.02
	(e) Dry rubble (rip-rap)	0.020-0.030
3	Tile	0.016-0.018
4	Brick	0.014-0.01
5	Sewers (concrete, A.C., vitrified-clay pipes)	0.012-0.01
6	Asphalt (1) Smooth	0.013
	(1) Rough	0.016
7	Concrete lined, excavated rock	
	(i) good section	0.017-0.02
	(II) Irregular section	0.022-0.02
8	Laboratory flumes-smooth metal bed and glass or perspex sides	0.009-0.01
(b)	Unlined, non-erodible channels	
1	Earth, straight and uniform	
	(1) clean, recently completed	0.016-0.02
	(II) clean, after weathering	0.018-0.02
	(III) gravel, uniform section, clean	0.022-0.03
	(iv) with short grass, few weeds	0.022-0.03
2	Channels with weeds and brush, uncut	
	(i) dense weeds, high as flow depth	0.05-0.12
	(II) clean bottom, brush on sides	0.04-0.08
	(III) dense weeds or aquatic plants in deep channels	0.03-0.035
	(iv) grass, some weeds	0.025-0.03
3	Rock	0.025-0.04
(0)	Natural channels	
1	Smooth natural earth channel, free from growth, little curvature	0.020
2	Earth channels, considerably covered with small growth	0.035
3	Mountain streams in clean loose cobbles, rivers with variable	0.04-0.05
	section with some vegetation on the banks	
4	Rivers with fairly straight alignment, obstructed by small trees, very little under brush	0.06-0.075
5	Rivers with irregular alignment and cross-section, covered with growth of virgin timber and occasional patches of bushes and small trees	0.125



Factors affecting Manning's n

- i. Surface roughness
- ii. Vegetation
- iii. Cross section irregularity
- iv. Irregular alignment of channels
- v. Silting & scouring
- vi. Obstruction
- vii. Stage & discharge
- viii. Seasonal change
- ix. Suspended material and bed load



Velocity Distribution

Wide channels

- i) Velocity Defect Law
- Channels with large aspect ratio B/y₀ are wide channels
 Flow is considered to be essentially two dimensional
 Velocity distribution is logarithmic



Fig. 3.3 Velocity profile in a wide open channel



Velocity Distribution Wide channels

The velocity u at a height y above the bed in a channel having uniform flow

at a depth y_0 is given by the velocity-defect law for $y/y_0 > 0.15$ as

$$\frac{u_m - u}{u_*} = \frac{1}{k} \ln \frac{y}{y_0}$$
$$= -\frac{2.3}{k} \log_{10} (y/y_0)$$

where $u_* =$ shear velocity = $\sqrt{\tau_0 / \rho} = \sqrt{gRS_0}$, R = hydraulic radius, $S_0 =$ longitudinal slope, and k = Karman constant = 0.41 for open channel flows⁵.



Velocity Distribution

Wide channels

in terms of the average velocity

$$V = \frac{1}{y_0} \int_0^{y_0} u \, dy \qquad \text{as}$$

$$u = V + \frac{u_*}{k} \left(1 + \ln \frac{y}{y_0} \right)$$

$$V = u_m - \frac{u_*}{k}$$

ii) Law of wall

$$\frac{u}{u_*} = \frac{1}{k} \ln \frac{yu_*}{v} + A_s$$

is found applicable in the inner wall region ($y/y_0 < 0.20$). The values of the constants are found to be k = 0.41 and $A_s = 5.29$ regardless of the Froude number and Reynolds number of the flow⁵. Further, there is an overlap zone between the law of the wall region and the velocity-defect law region.



Velocity Distribution

For completely rough turbulent flows, the velocity distribution in the wall region $(y/y_0 < 0.20)$ is given by u = 1 by y = -4

$$\frac{u}{u_*} = \frac{1}{k} \ln \frac{y}{\varepsilon_s} + A_i$$

where ε_s = equivalent sand grain roughness. It has been found that *k* is a universal constant irrespective of the roughness size⁵. Values of *k* = 0.41 and A_r = 8.5 are appropriate.

Channels with small Aspect Ratio

channels with $B/y_0 \le 5$ can be classified as *narrow channels*.



Velocity Distribution Channels with small Aspect Ratio



Fig. 3.4 Velocity profile in a narrow channel





Fig. 3.5(a) Typical velocity distribution in a narrow channel, $B/y_0 = 1.0$.





Fig. 3.5(b) Typical velocity distribution in a rectangular channel with $B/y_0 = 6.0$. (Ref.47)



n varies from 0.01 (smooth metal frame) to 0.2 (natural channels with irregular sections)



Equivalent roughness will be between 0.01 and 0.015



Horton's Method of Equivalent Roughness



Fig. 3.10 Multi-roughness type perimeter

ula
$$V_i = \frac{1}{n} R_i^{2/3} S_i^{1/2}$$

Manning's Formula



Assume all the mean velocities are same for all sections and channel slope is also same

$$S = \frac{Vn_{eq}}{R^{2/3}} = \frac{V_i n_i}{R_i^{2/3}}$$

All Vs are same and cancelled







developed by Horton in 1933 and by Einstein in 1934. However, Eq. 3.26 is popularly known as Horton's formula



Sl. No	Investigator	n _e	Concept
1	Horton (1933); Einstein (1934)	$= \left[\frac{1}{P} \sum (n_i^{3/2} P_i)\right]^{2/3}$	<i>Mean velocity is constant in all subareas.</i>
2	Pavlovskii (1931) Muhlhofer (1933) Einstein and Banks (1950)	$= \left[\frac{1}{P}\sum_{i=1}^{2} (n_i^2 P_i)\right]^{1/2}$	Total resistance force F is sum of subarea resistance forces, $\sum F_i$
3	Lotter (1932)	$=\frac{PR^{5/3}}{\sum \frac{P_i R_i^{5/3}}{n_i}}$	Total discharge is sum of subarea discharges

(Continued)



Example 3.3 An earthen trapezoidal channel (n = 0.025) has a bottom width of 5.0 m, side slopes of 1.5 horizontal: 1 vertical and a uniform flow depth of 1.1 m. In an economic study to remedy excessive seepage from the canal two proposals, viz (a) to line the sides only, and (b) to line the bed only are considered. If the lining is of smooth concrete (n = 0.012), determine the equivalent roughness in the above two cases by using (i) Horton's formula, and by (ii) Pavlovskii formula.

Solution Case (a) Lining of the sides only Here for the bed: $n_1 = 0.025$, and $P_1 = 5.0$ m.

For the sides: $n_2 = n_3 = 0.012$, and $P_2 = P_3 = 1.10 \times \sqrt{1 + (1.5)^2} = 1.983$ m

(i) Equivalent roughness n_e by Horton's formula: $n_e = \left[\frac{1}{P}\sum_{i=1}^{2} (n_i^{3/2}P_i)\right]^{2/3}$

$$n_{e} = \frac{\left[5.0 \times (0.025)^{3/2} + 1.983 \times (0.012)^{3/2} + 1.983 \times (0.012)^{3/2}\right]^{2/3}}{\left[5.0 + 1.983 + 1.983\right]^{2/3}} = \frac{0.085448}{4.31584} = 0.0198$$



Case (b) Lining of the bed only Here for the bed: $n_1 = 0.012$ and $P_1 = 5.0$ m.

For the sides: $n_2 = n_3 = 0.025$, and $P_2 = P_3 = 1.10 \times \sqrt{1 + 1.5}^2 = 1.983$ m

(i) Equivalent roughness n_{e} by Horton's formula: $n_{e} = \left[\frac{1}{P}\sum_{i}(n_{i}^{3/2}P_{i})\right]^{2/3}$

$$n_{\rm e} = \frac{\left[5.0 \times (0.012)^{3/2} + 1.983 \times (0.025)^{3/2} + 1.983 \times (0.025)^{3/2}\right]^{2/3}}{\left[5.0 + 1.983 + 1.983\right]^{2/3}} = \frac{0.079107}{4.31584} = 0.01833$$



(ii) Equivalent roughness n_e by Pavlovskii formula: $n_e = \left[\frac{1}{P}\sum_{i=1}^{n_e} (n_i^2 P_i)\right]^{n_e}$

$$n_e = \frac{\left[5.0 \times (0.012)^2 + 1.983 \times (0.025)^2 + 1.983 \times (0.025)^2\right]^{1/2}}{\left[5.0 + 1.983 + 1.983\right]^{1/2}} = \frac{0.05656}{2.99433} = 0.01889$$



LANE J.3 (Continued)

Sl. No	Investigator	n _e	Concept
4	Felkel (1960)	$=\frac{P}{\sum \frac{P_i}{n_i}}$	Total discharge is sum of subarea discharges
5	Krishnamurthy and Christensen (1972)	$= \exp\left[\frac{\sum P_i y_i^{3/2} \ln n_i}{\sum P_i y_i^{3/2}}\right]$	Logarithmic velocity distribution over depth y _i for wide channel
6	Yen (1991)	$=\frac{\sum (n_i P_i)}{P}$	<i>Total shear velocity is weighted sum of subarea shear velocity</i>



Manning's formula and the continuity equation, Q = AVThe discharge Q is then given by $Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$ $= K \sqrt{S_0}$

where, $K = \frac{1}{n} A R^{2/3}$ is called the *conveyance* of the channel and expresses the dis-

charge capacity of the channel per unit longitudinal slope.

The term $nK = AR^{2/3}$ is

sometimes called the section factor for uniform-flow computations.

Section Factor Z = nK



For a given channel, $AR^{2/3}$ is a function of the depth of flow.

a trapezoidal section of bottom width = B and side slope *m* horizontal: 1 vertical. Then,

$$A = (B + my)y$$
$$P = (B + 2y \sqrt{m^{2} + 1})$$
$$R = \frac{(B + my)y}{(B + 2y \sqrt{m^{2} + 1})}$$
$$AR^{2/3} = \frac{(B + my)^{5/3}y^{5/3}}{(B + 2y \sqrt{m^{2} + 1})^{2/3}} = f(B, m, y)$$

For a given channel, *B* and *m* are fixed and $AR^{2/3} = f(y)$.





Fig. 3.9 Variation of ϕ with y/B in trapezoidal channels



For a rectangular section

$$Z = \frac{B^{5/3} y^{5/3}}{(B+2y)^{2/3}}$$





For very wide rectangular channel (B>20y) $Z = By^{5/3}$







It may be seen that for $m \ge 0$, there is only one value y/B for each

value of ϕ , indicating that for $m \ge 0$, $AR^{\nu/3}$ is a single-valued function of y.

top width is either constant or increases with depth.

channels of the first kind.

Since $AR^{\nu/3} = \frac{Qn}{\sqrt{S_0}}$ and if *n* and S_0 are fixed for a channel, the channels of the first kind have a unique depth in uniform flow associated with each discharge. This depth is called the *normal depth*.

The normal depth is designated as y_0 ,

The channels of the first kind thus have one normal depth only.



Channels with a closing top-width can be designated as *channels of the second kind*.

Example 3.7 A 5.0-m wide trapezoidal channel having a side slope of 1.5 horizontal: 1 vertical is laid on a slope of 0.00035. The roughness coefficient n = 0.015. Find the normal depth for a discharge of 20 m³/s through this channel.

Let				
	$y_0 = $ normal depth			
Area	$A = (5.0 + 1.5 y_0) y_0$			
Wetted perimeter	$P = 5.0 + 2\sqrt{3.25} y_0$			
	$= 5.0 + 3.606 y_0$			
	$R = A/P = \frac{(5.0 + 1.5y_0)y_0}{(5.0 + 3.606y_0)}$			
The section factor $AR^{\nu/3} = \frac{Qn}{\sqrt{S_0}}$				
$\frac{(5.0+1.5y_0)^{5/3}y_0^{5/3}}{(5.0+3.606y_0)^{2/3}} = \frac{20 \times 0.015}{(0.00035)^{1/2}} = 16.036$				

Algebraically, y_0 can be found from the above equation by the trial-and-error method. The normal depth is found to be 1.820 m



SECOND HYDRAULIC EXPONENT N

$$K^2 = C_2 y^N$$

Where C_2 = a coefficient and N = an exponent called here as the *second hydraulic*

Example 3.17 Obtain the value N for (a) a wide rectangular channel, and (b) a triangular channel.

(a) For a Wide Rectangular Channel Considering unit width, A = y

$$R = y$$

$$K^{2} = \frac{1}{n^{2}} y^{2} (y^{4/3}) = C_{2} y^{N}$$

By equating the exponents of *y* on both sides; N = 3.33

(b) For a Triangular Channel of Side Slope *m* Horizontal: 1 Vertical $A = my^{2}, P = 2y\sqrt{m^{2} + 1}$ $R = \frac{m}{2\sqrt{m^{2} + 1}}y$ $K^{2} = \frac{1}{n^{2}}(my^{2})^{2} \left(\frac{m}{2\sqrt{m^{2} + 1}}y\right)^{4/3} = C_{2}y^{N}$ By equating

By equating the exponents of *y* on both sides, N = 5.33.







HYDRAULICALLY EFFICIENT CHANNEL SECTION

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}$$

For a given area
$$\longrightarrow$$
 Minimum Perimeter
For a given Perimeter \longrightarrow Maximum Area

For maxima or minima we have to take $\frac{d}{dy}$ of the parameter = 0 and then go for the second derivative

So the condition for maximum discharge will be $\frac{dQ}{dv} = 0$

Or we can write

$$\frac{d}{dy}(AK^{5} / P^{2}) = 0$$

$$\frac{d}{dy}(A^{5} / P^{2}) = 0$$

 $d_{(AD^{2/3})}$ 0



HYDRAULICALLY EFFICIENT CHANNEL SECTION

When slope, roughness coefficient, area of flow are fixed

Then minimum perimeter section will represent Hydraulically Efficient Section as it conveys the maximum discharge also called Best Section

(a) Rectangular Section Bottom width = B and depth of flow = y Area of flow A = By = constantWetted perimeter $P = B + 2y = \frac{A}{y} + 2y$

If P is to be minimum with A

= constant,



Fig. 3.18 Hydraulically efficient rectangular channel



HYDRAULICALLY EFFICIENT CHANNEL SECTION

Triangular Section



eferring to Fig. 3.23, from Eq. (3.33), the area of flow section

$$\mathbf{A} = \frac{D^2}{8} \left(2\theta - \sin 2\theta \right)$$

and from Eq. (3.34), the wetted perimeter $P = D\theta$

For the maximum discharge, from Eq. (3.49a),

$$\frac{d}{d\theta} \left(A^5 / P^2 \right) = 0$$

i.e.

$$5P\frac{dA}{d\theta} - 2A\frac{dP}{d\theta} =$$



Fig. 3.23 Circular channel

$$5D\theta \frac{D^2}{8}(2 - 2\cos 2\theta) - 2\frac{D^2}{8}(2\theta - 2\sin 2\theta)D = 0$$

$$3\theta - 5\theta\cos 2\theta + \sin 2\theta = 0$$



PROPERTIES OF SOME MOST EFFICIENT CHANNEL SECTIONS

 Table 3.4 Proportions of some most efficient sections

SL. No	Channel Shape	Area (A _{em})	Wetted Perimeter (P _{em-})	Width (B _{em})	Hydraulic Radius (R _{em})	Top width (T _{em})	$\frac{Qn}{\overline{y_{em}^{8/3}S_0^{1/2}}} = K_{em}$
1	Rectangle (half square)	$2 y_{em}^2$	4 y _{em}	2 y _{em}	$\frac{y_{em}}{2}$	2 y _{em}	1.260
2	Trapezoidal (half regular hexagon, $m = \frac{1}{\sqrt{3}}$)	$\sqrt{3}y_{em}^2$	$2\sqrt{3}y_{em}$	$\frac{2}{\sqrt{3}}y_{em}$	<u>Y_{em}</u> 2	$\frac{4y_{em}}{\sqrt{3}}$	1.091
3	Circular (semicircular)	$\frac{\pi}{2} y_{em}^2$	$\pi y_{_{em}}$	$D = 2y_{em}$	$\frac{y_{em}}{2}$	2 y _{em}	0.9895
4	Triangle (Vertex angle = 90º	y_{em}^2	$2\sqrt{2}y_{em}$	_	$\frac{y_{em}}{2\sqrt{2}}$	2 y _{em}	0.500



SECOND HYDRAULIC EXPONENT N

Conveyance K of a channel is function of depth of flow y

Bakhmeteff gave the equation $K^2 = C_2 y^N$

Where C_2 = a coefficient and N = an exponent called here as the *second hydraulic* exponent

first hydraulic exponent M associated with the critical depth.

Example 3.17 Obtain the value N for (a) a wide rectangular channel, and (b) a triangular channel.

(a) For a Wide Rectangular Channel Considering unit width, A = y (b) For a Triangular Channel of Side Slope *m* Horizontal: 1 Vertical $A = my^2$, $P = 2y\sqrt{m^2 + 1}$

 $R = \frac{m}{2\sqrt{m^2 + 1}} y$

$$R = y$$

$$K^{2} = \frac{1}{n^{2}} y^{2} (y^{4/3}) = C_{2} y^{N}$$

By equating the exponents of y on both sides; N = 3.33

$$K^{2} = \frac{1}{n^{2}} (my^{2})^{2} \left(\frac{m}{2\sqrt{m^{2}+1}}y\right)^{4/3} = C_{2}y^{N}$$
$$N = 5.33.$$



COMPOUND CHANNELS





Fig. 3.30 Channel cross-sectional area division for diagonal interface procedure—Example—3.23



COMPOUND CHANNELS

At $y = y_m$ main channel depth of flow

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

At y = y_m +
$$\Delta y$$
 $Q_{m+\Delta m} = \frac{1}{n} A R_1^{2/3} S^{1/2}$

P increases too much in flood plain than the area A so R reduces

So $Q_{m+\Delta m}$ may be smaller than Q because R is now reduced but physically this is not correct

$$Q_{Total} = \frac{1}{n} A R^{2/3} S^{1/2} \qquad \qquad Q_{Partial} = Q_1 + Q_2 + \dots Q_3$$

Choose maximum value



TODAY'S DEAL

ASSIGNMENT -2

SOLVE ANY 10 UNSOLVED QUESTIONS FROM THE CHAPTER : UNIFORM FLOW

LAST DATE: 27th August 2021, FRIDAY

MAIL TO : hhmc2021@gmail.com



THE END