*Subject Code: NCE 202*

*Subject Name: Hydraulics & Hydraulic Machines*

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# Syllabus
# Subject Name: Hydraulics & Hydraulic Machines

**HYDRAULICS AND HYDRAULIC MACHINES (ECE 301)**

**L T P C 2 1 2 4**

**Unit-I**

Introduction: Difference between pipe flow and open channel flow. Types of open channels, Types of flows in open channel, Geometric elements, Velocity distribution, Velocity and pressure distribution in an open channel, Continuity equation.

Uniform Flow: Chezy's& Manning's formula, Roughosity coefficient, Uniform Flow computations, Hydraulically efficient section (Rectangular, Triangular, Trapezoidal ), compound channel sections.

**Unit-II**

Depth energy relationship in open channel flow:Specific energy (definition & diagram, Critical, Sub-critical, Super-critical flow), Specific force, Specific discharge, flow through vertical and horizontal contractions.

**Unit-III**

Gradually varied flow (G.V.F.):Definition, Classification of channel Slopes, Dynamic equation of G.V.F.( Assumption and derivation), Classification of G.V.F. profiles-examples, Direct step method of Computation of G.V.F. profiles.

# Syllabus

# Subject Name: Hydraulics & Hydraulic Machines

**Unit-IV**

Rapidly varied flow (R.V.F.):Definition, examples, Hydraulic jump- Phenomenon, relation of conjugate depths, Parameters, Uses, Types of Hydraulic jump, Hydraulic jump as an energy dissipater, Notches & Weirs : Types, derivation of discharge equation, Sharp, broad & round crested weirs.

**Unit-V**

Impact of jet: Impulse momentum principle, Impact of jet on Vanes-flat, curved (stationary and moving), Inlet & outlet velocity triangles, Series of flat, curved vanes mounted on wheel.
Hydraulic turbines:Importance of hydro-power, Classification of turbines, description, Typical dimensions and working principle of Pelton, Francis & Kaplan turbine, Unit quantities, Specific speed, Performance Characteristics, Selection of type of turbine, description & function of Draft tube

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# Subject Name: Hydraulics & Hydraulic Machines

## List of Experiments

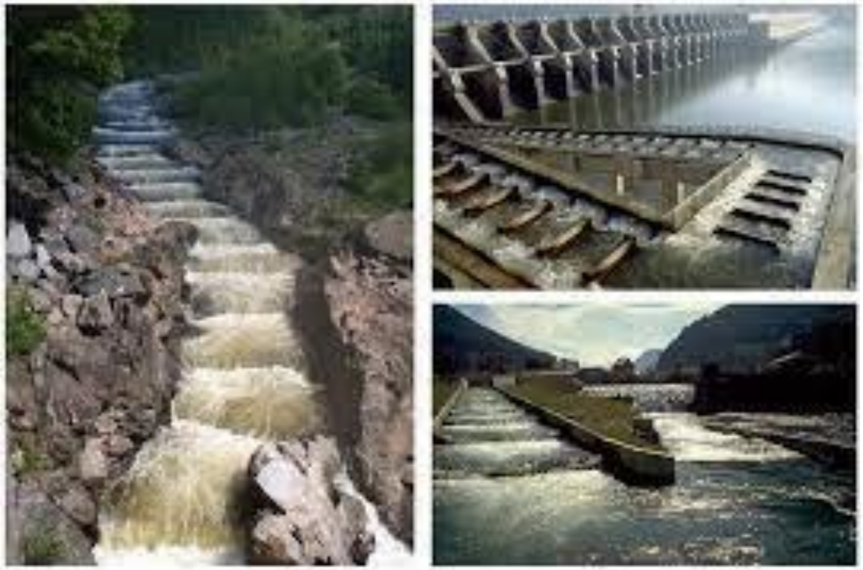
1. To determine the Manning's coefficient of roughness 'n' for the given channel bed.
2. To study the velocity distribution in an open channel and to find the energy and momentum correction factors.
3. To study the flow characteristics over a hump placed in an open channel.
4. To study the flow through a horizontal contraction in a rectangular channel.
5. To calibrate a broad-crested weir and sharp crested spillway.
6. To study the characteristics of free hydraulic jump.
7. To study the flow over an abrupt drop and to determine the end (brink) depth for a free over fall in an open channel.
8. To study rotodynamic pumps and their characteristics.
9. To study rotodynamic turbines and their characteristics.
10. To calibrate and to determine the coefficient of discharge for rectangular and triangular notches.
11. To verify the momentum equation.

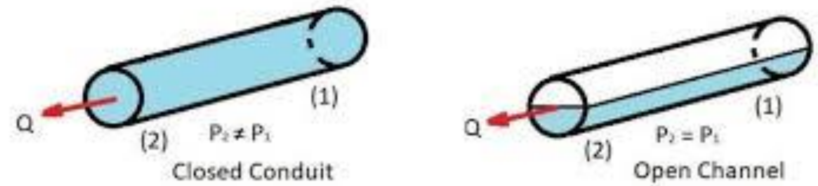
# Syllabus
# Subject Name: Hydraulics & Hydraulic Machines

**References:**

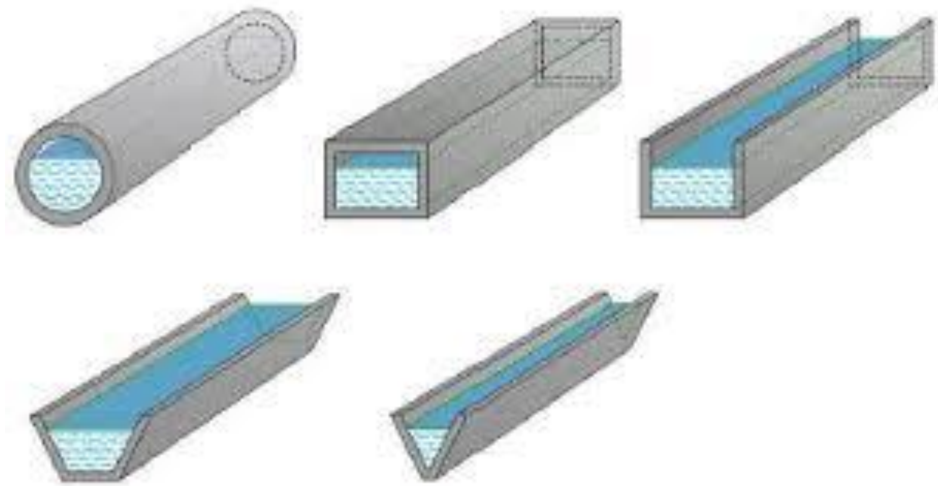
1. Subramanya, K., Flow in Open Channels, Tata McGraw Hill
2. Srivastava R., Flow through open channel, Oxford university press.
2. Henderson, F.M., Open Channel Flow, McGraw Hill International
3. Chow, V.T., Open channel Hydraulics, McGraw Hill International
4. Ranga Raju, K.G., Flow through open channels, T.M.H.
5. French, R.H., Open Channel Hydraulics, McGraw Hill International
6. Graf, W.H., Hydraulics of Sediment Transport, McGraw Hill International
7. Fluid Mechanics – K. Subramanyam – Tata McGraw-Hill Pub. Co., Delhi
8. Fluid Mechanics – Hydraulic & Hydraulic Mechanics -Modi / seth – Standard Book House Delhi

# 1 Introduction



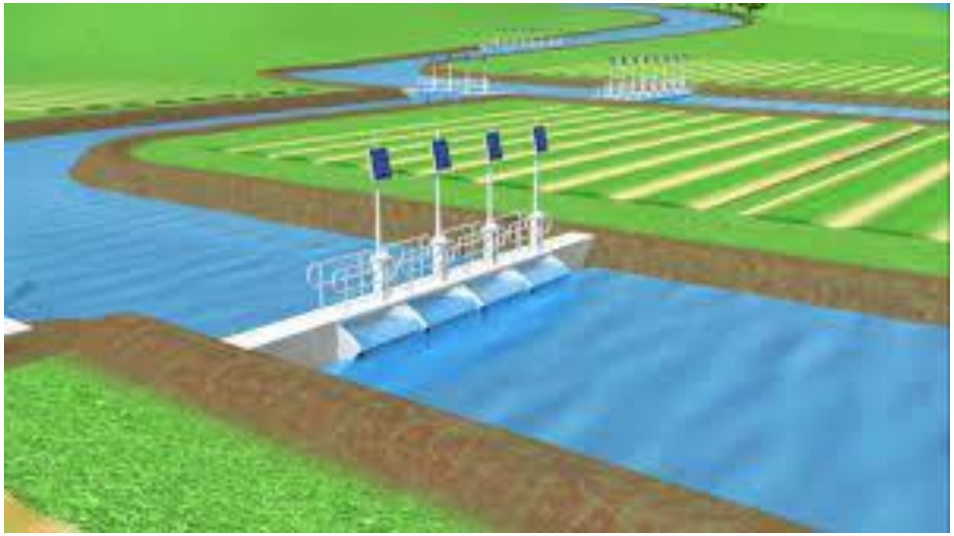

Q
(1)
(2)
$P_2 \neq P_1$
Closed Conduit
Q
(1)
(2)
$P_2 = P_1$
Open Channel


OPEN CHANNEL FLOW



IRRIGATION


## CONTROLLING MECHANISM



RIVER FLOW

DAMS

SEWERAGE SYSTEM


BARRAGE

- Open channel flow is a flow which has a free surface and flows due to gravity.
- Pipes not flowing full also fall into the category of open channel flow
- In open channels, the flow is driven by the slope of the channel rather than the pressure

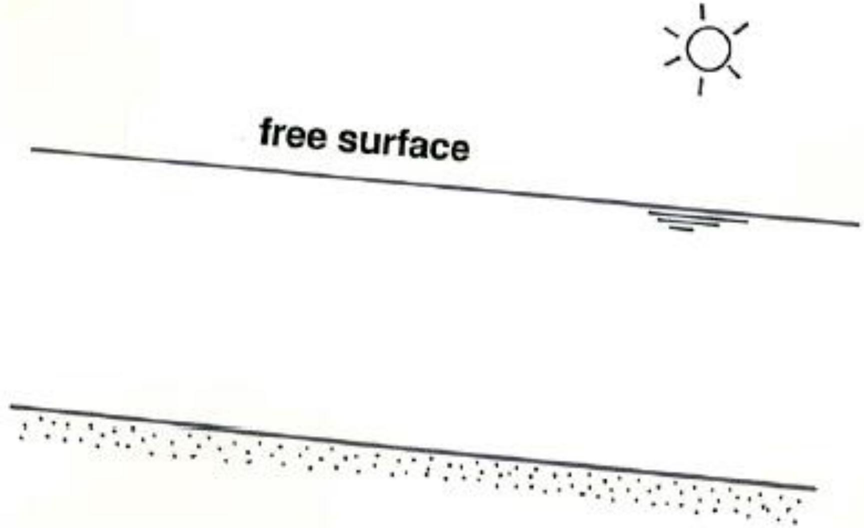



Figure 5-1. An open-channel flow.

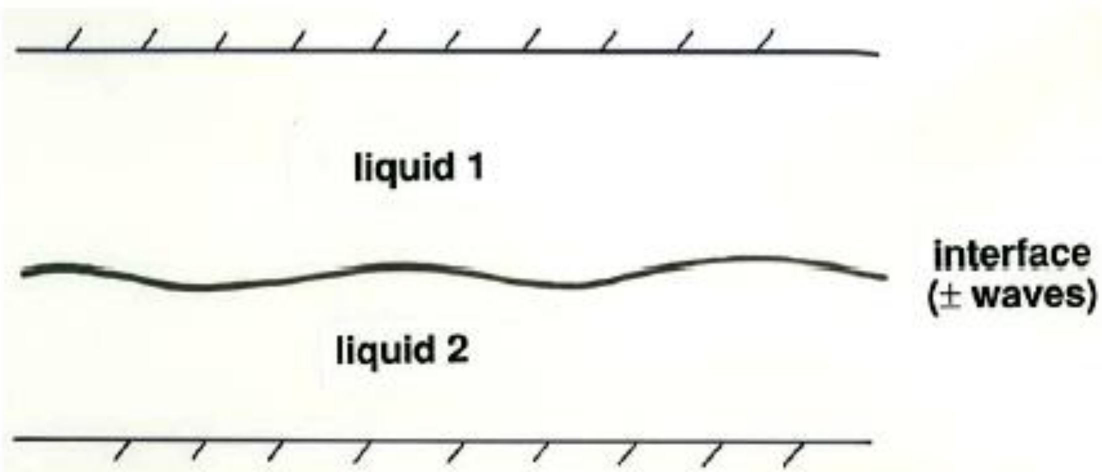



Figure 5-2. A free-surface flow that is not an open-channel flow.

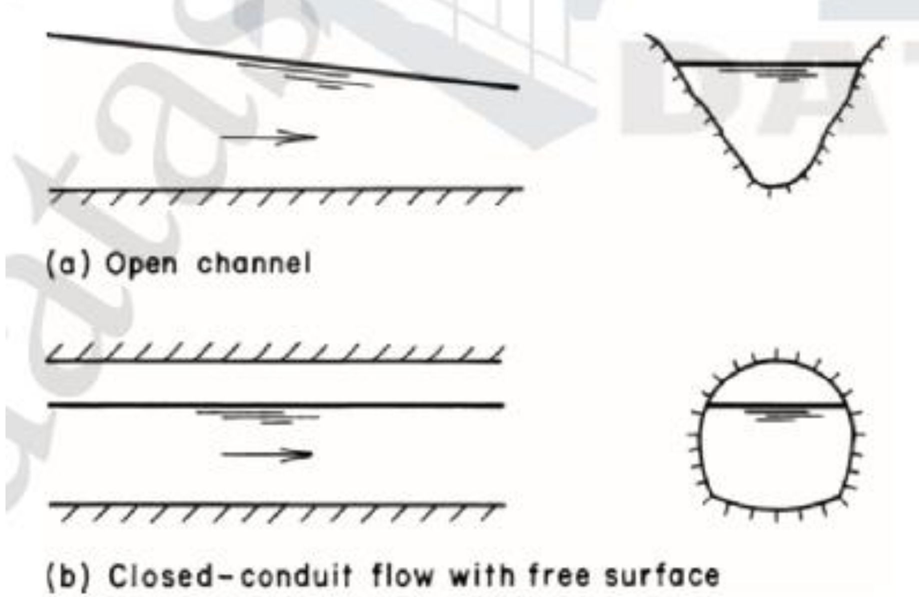



Fig. 1-1. Free-Surface flow

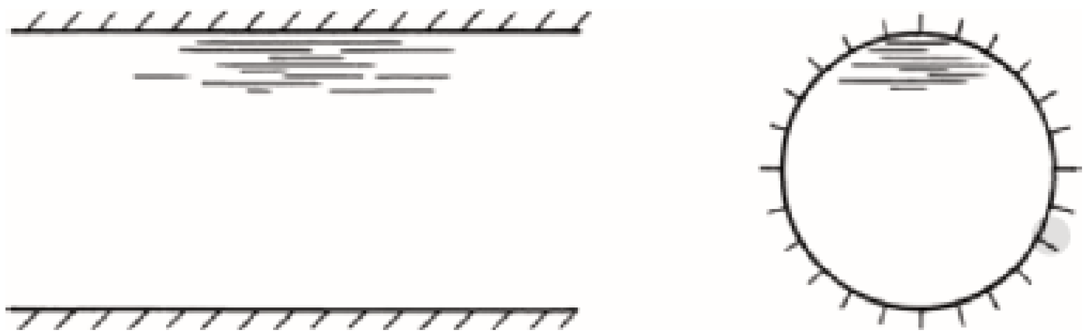
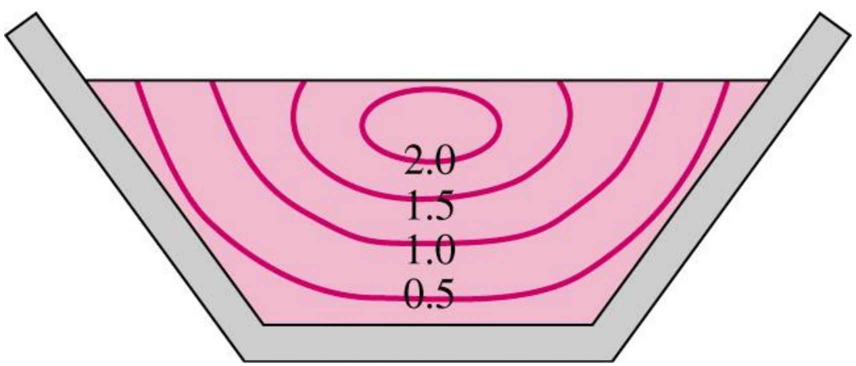


Fig. 1-2. Pipe or pressurized flow

# Classification of Open-Channel Flows



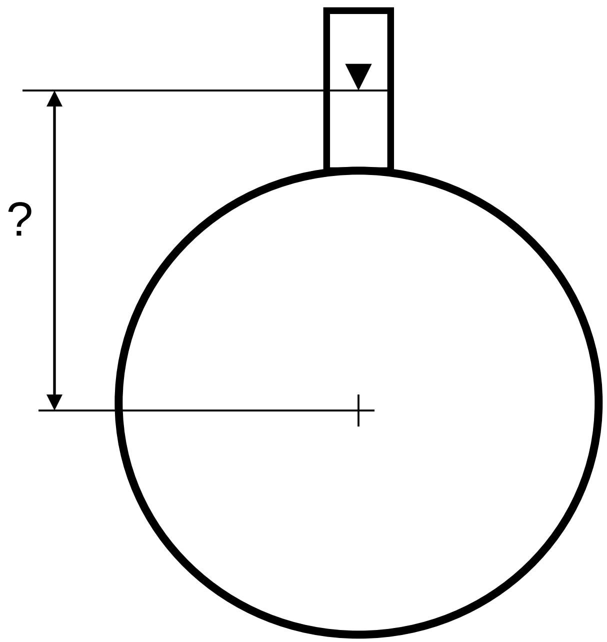

- Open-channel flows are characterized by the presence of a liquid-gas interface called the *free surface.*
- Natural flows: rivers, creeks, floods, etc.
- Human-made systems: fresh-water aqueducts, irrigation, sewers, drainage ditches, etc.

# DIFFERENCES BETWEEN PIPEFLOW AND OPEN CHANNEL FLOW

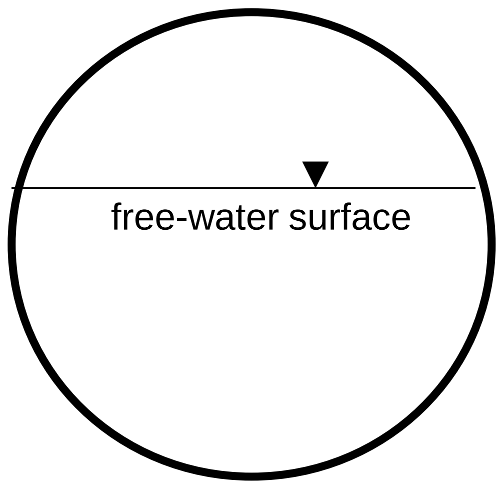
**Table 1.1** *Essential differences between pipe flow and open-channel flow*

| No. | Pipe Flow | Open Channel Flow |
|---|---|---|
| 1. | No free surface, pipe always runs full. | Presence of free surface is the essential feature of open channel flow. |
| 2. | Flow due to energy gradient | Flow due to potential energy gradient (i.e. component of gravity) |
| 3. | Hydraulic grade line, i.e., piezometric head $\left(\frac{p}{\gamma}+Z\right)$ line, can be above or below the pipe axis. | Hydraulic grade line coincides with the free surface. |
| 4. | For a given pipe, the flow takes place in a pre-fixed cross section. | In a given channel carrying a given discharge, the depth of flow can vary both in time and along the channel depending upon the nature of the flow. The flow section area is a variable. |
| 5. | The pipe cross section is of fixed geometry. A majority of pipes used are of circular cross section and the size range is in a reasonably large range. | The depth of flow can vary in a given flow. The cross-sectional shapes and sizes, especially in natural channels, are in an extremely wide range covering several orders of magnitude. |
| 6. | The boundary roughness (expressed as relative roughness) is within a fairly reasonable range of a few orders of magnitude. | Roughness magnitudes encountered on open channels cover an extremely wide range covering several orders of magnitude. |
| 7. | Reynolds number, representing the influence of fluid viscosity, is the prime non-dimensional number governing the flow. | Froude number, representing the influence of gravity, is the prime non-dimensional number governing the flow. |

# Pressure vs. Open-channel flow




pressure flow
(pressure driven)




open-channel flow
(typ. gravity driven)

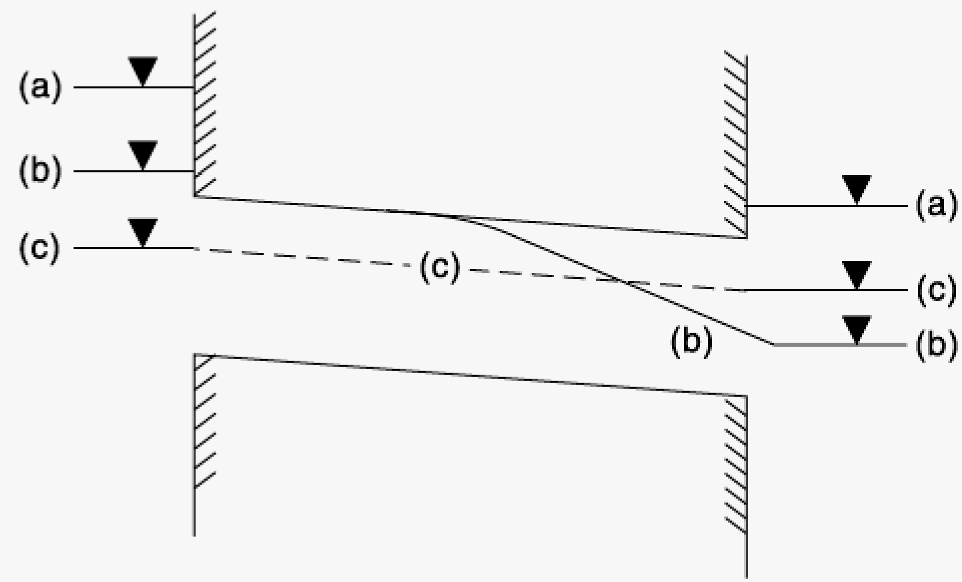



**Fig. 1.1** Flow through a culvert: (a) closed conduit or pressure flow, (b) partly closed and partly open flow, and (c) open channel or free surface flow

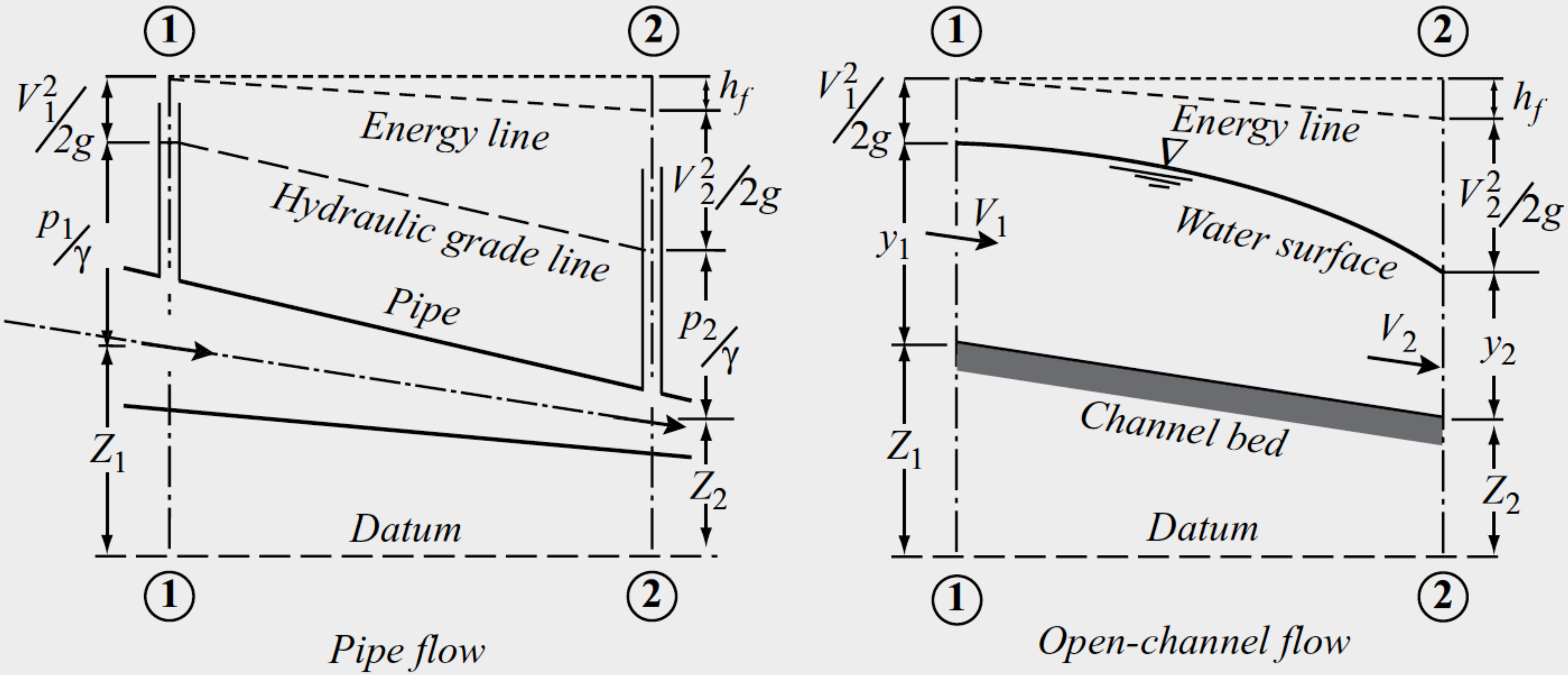



**Fig. 1.1** *Schematic representation of pipe flow and open-channel flow*

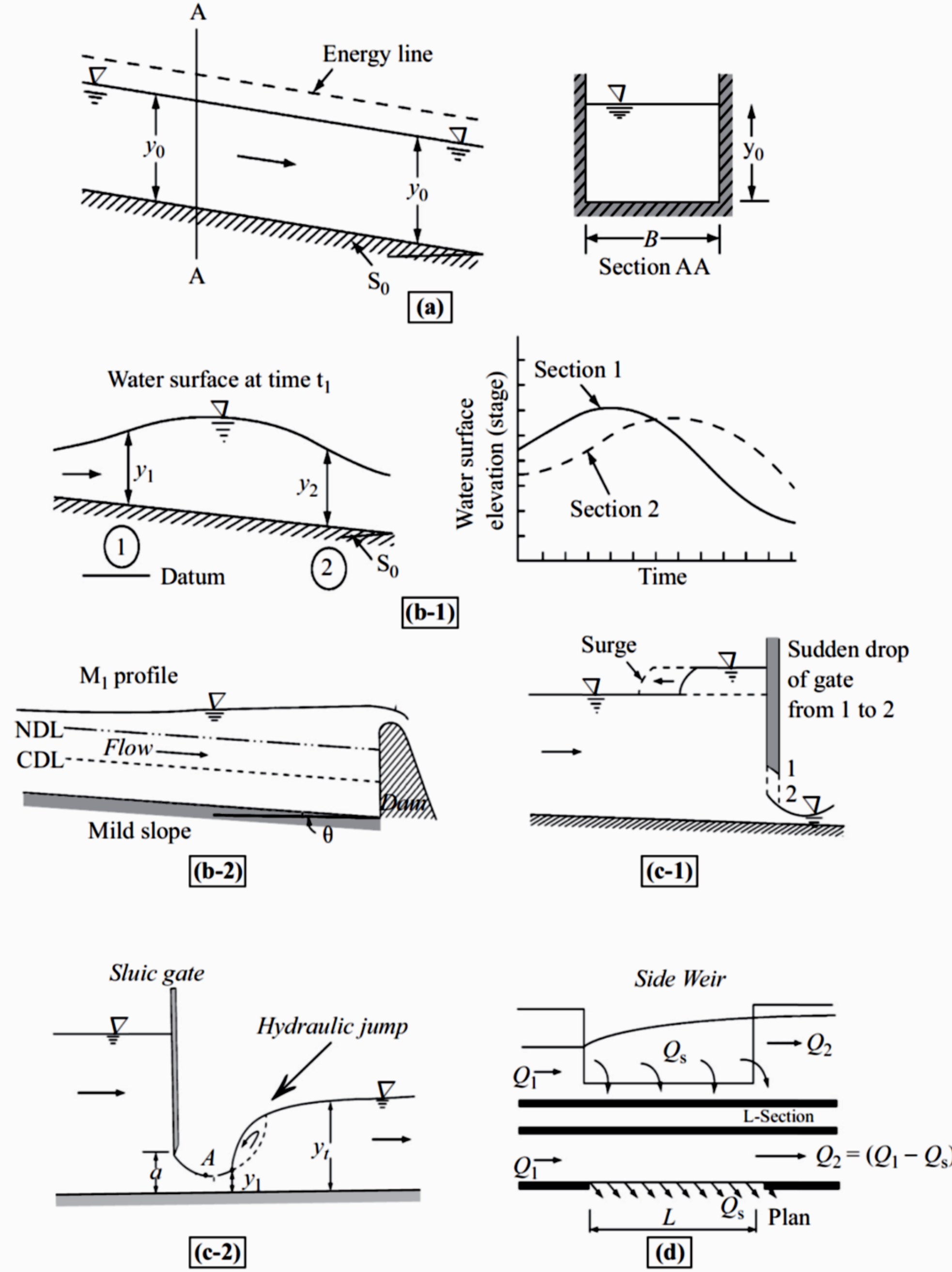



**Fig. 1.2** *Various types of open-channel flows: (a) Uniform flow (b-1) Unsteady GVF, (b-2) Steady GVF (c-1) Unsteady RVF, (c-2) Steady RVF (d) Spatially varied flow*

# Section Parameters

- **Reach Length** =L [L]= arbitrary horizontal distance between two cross-sections
- **Bed slope** = So [L/L] = -dz/dx = -(z2-z1)/L = $\tan\theta$
- **Flow depth** =y [L] = vertical distance from channel bottom to free water surface
- **Depth of flow sec** =d [L] = flow depth normal to flow, $d = y\cos\theta$
- **Top width** = T[L] = Width at free-surface
- **Flow area** = A[L2] = cross-sectional area normal to flow direction
- **Wetted perimeter** = P[L] = length of channel boundary in contact with water
- **Hydraulic radius** = R[L] = A/P
- **Hydraulic depth** = D[L] = A/T
- **Section Factor Critical Flow** = Zc $= AD^{1/2}$ $[L^{5/2}]$
- **Section Factor Uniform Flow** = Zn = $AR^{2/3}$ $[L^{8/3}]$

# Section Parameters

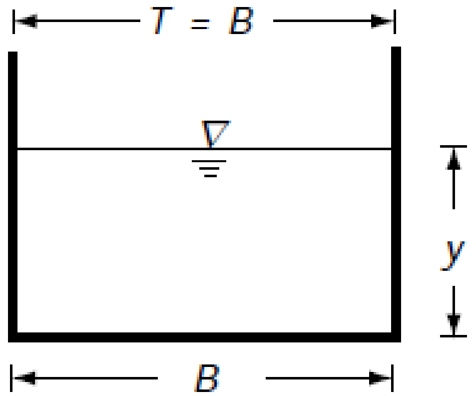


Fig. 2.3 *Rectangular channel*

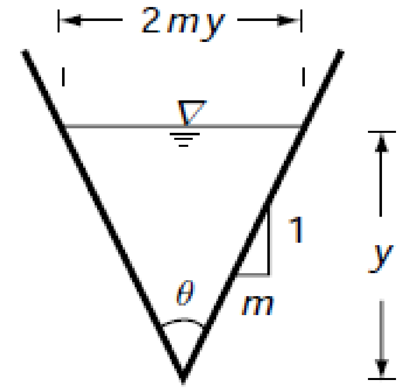


Fig. 2.4 *Triangular channel*

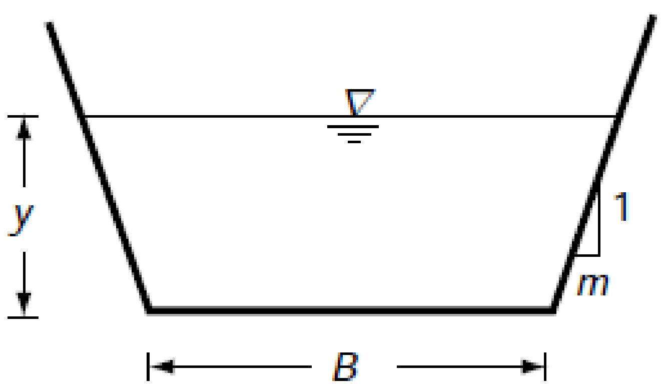


Fig. 2.6 *Trapezoidal channel*

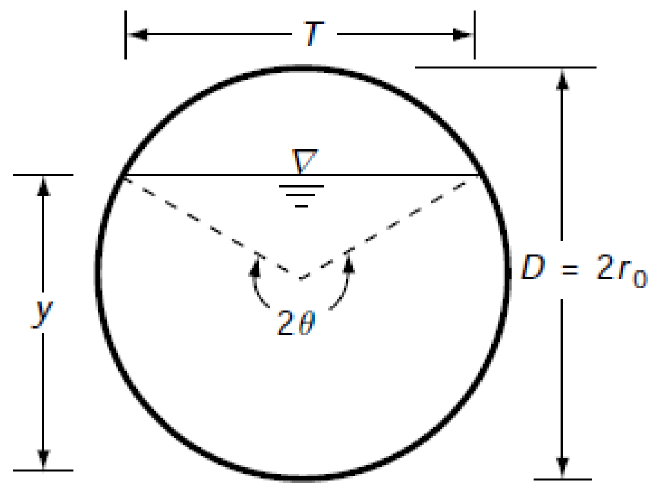
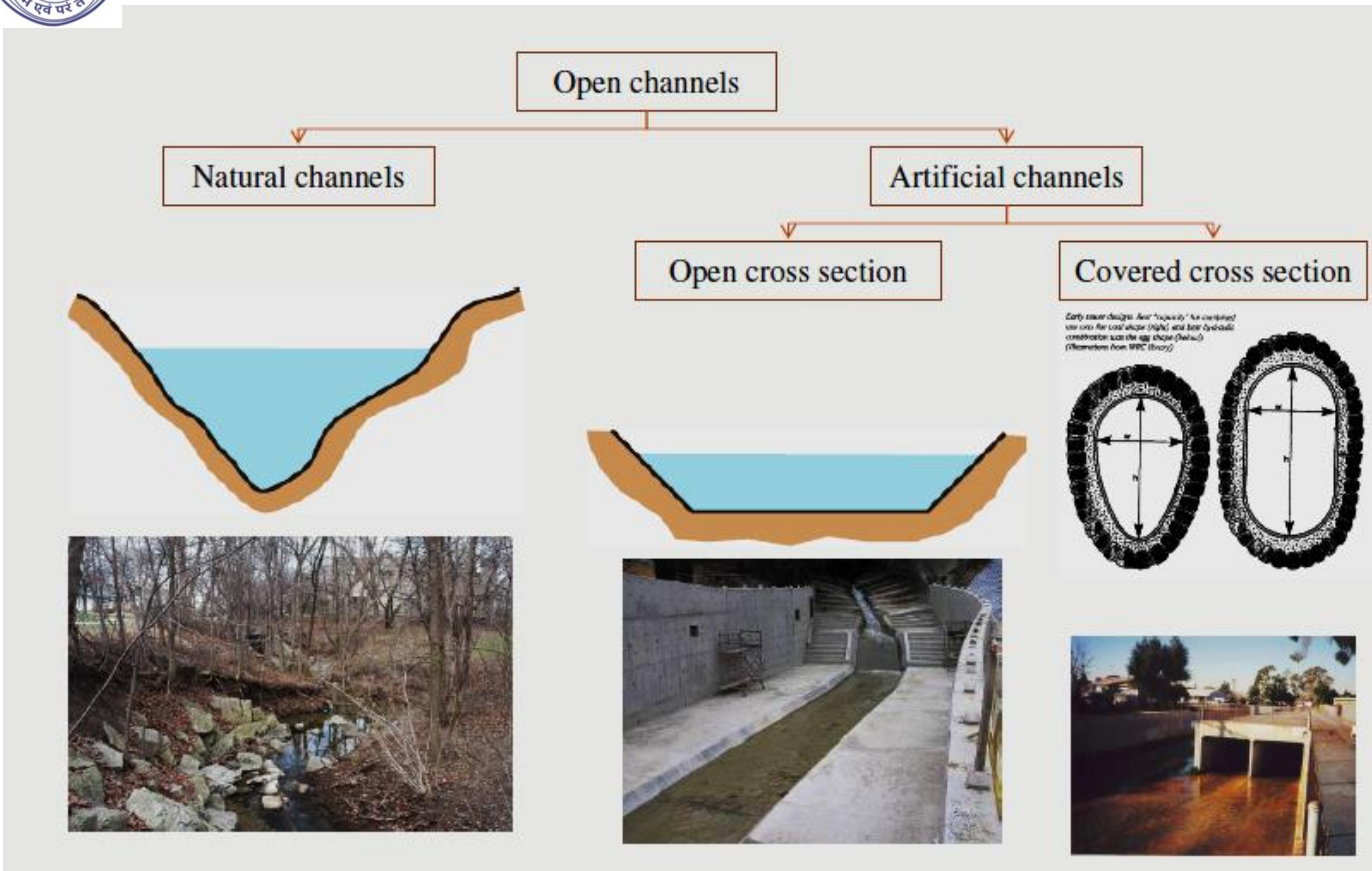


Fig. 2.5 *Circular channel*

# KINDS OF OPEN CHANNELS



# KINDS OF OPEN CHANNELS

- Canal
- Flume
- Chute
- Drop
- Culvert
- Open-Flow Tunnel

# KINDS OF OPEN CHANNELS

- CANAL is usually a long and mild-sloped channel built in the ground.

## KINDS OF OPEN CHANNELS

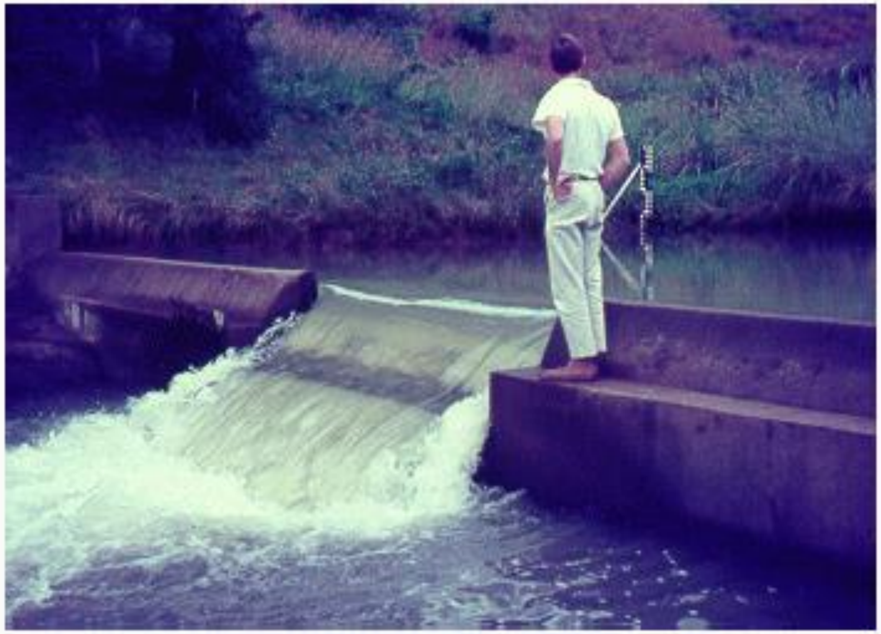
- FLUME is a channel usually supported on or above the surface of the ground to carry water across a depression.

# KINDS OF OPEN CHANNELS

- CHUTE is a channel having steep slopes.

## KINDS OF OPEN CHANNELS

- DROP is similar to a chute, but the change in elevation is affected in a short distance.



# Kinds of Open Channel

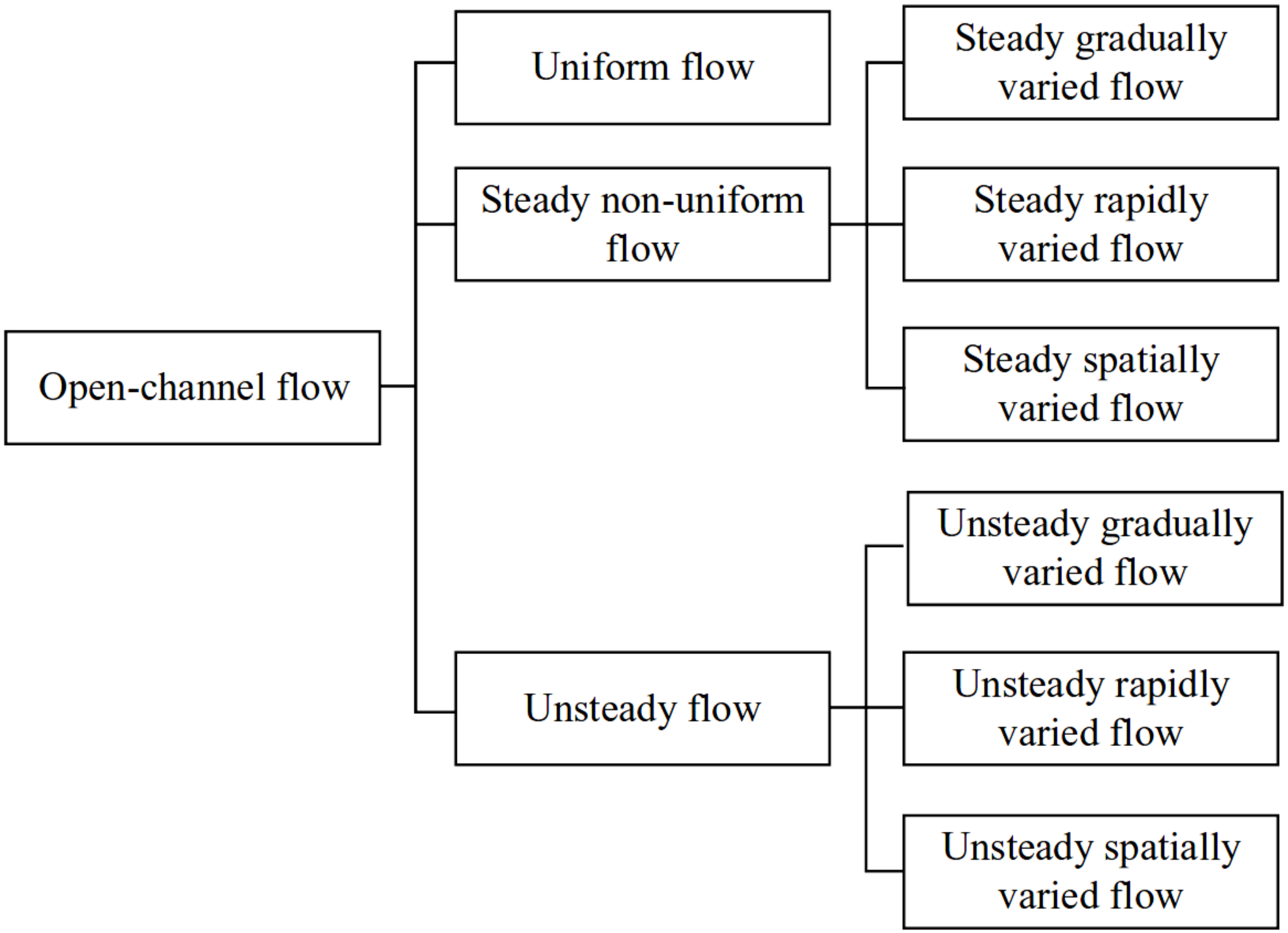
- CULVERT is a covered channel flowing partly full, which is installed to drain water through highway and railroad embankments.

## KINDS OF OPEN CHANNELS

- OPEN-FLOW TUNNEL is a comparatively long covered channel used to carry water through a hill or any obstruction on the ground.

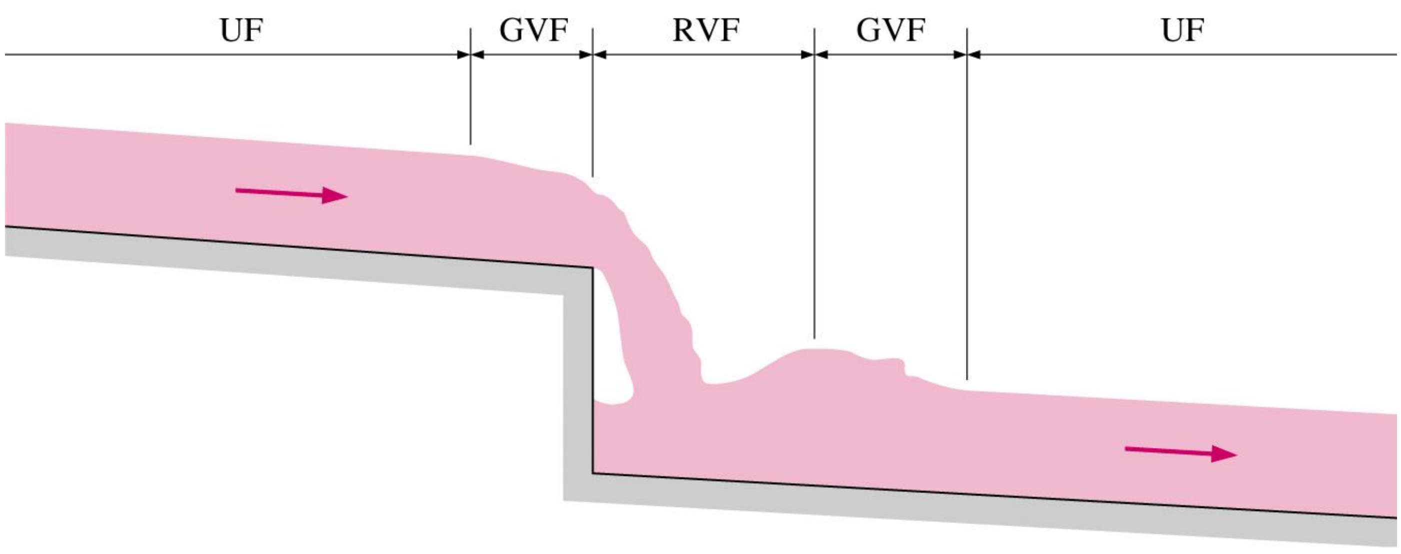
# CLASSIFICATION OF OPEN CHANNEL FLOWS

Thus, open-channel flows are classified for purposes of identification and analysis as follows:



# Classification of Open-Channel Flows

- Obstructions cause the flow depth to vary.
- Rapidly varied flow (RVF) occurs over a short distance near the obstacle.
- Gradually varied flow (GVF) occurs over larger distances and usually connects UF and RVF.



# Classification of Open-Channel Flows

## Classification Based on Channel Characteristics

- **PRISMATIC AND NONPRISMATIC CHANNELS:**

A channel is said to be *prismatic* if it is in the form of a prism, i.e., the cross section and the bed slope do not change along the channel length. If there is a change in cross section and/or slope, the channel is *nonprismatic*.

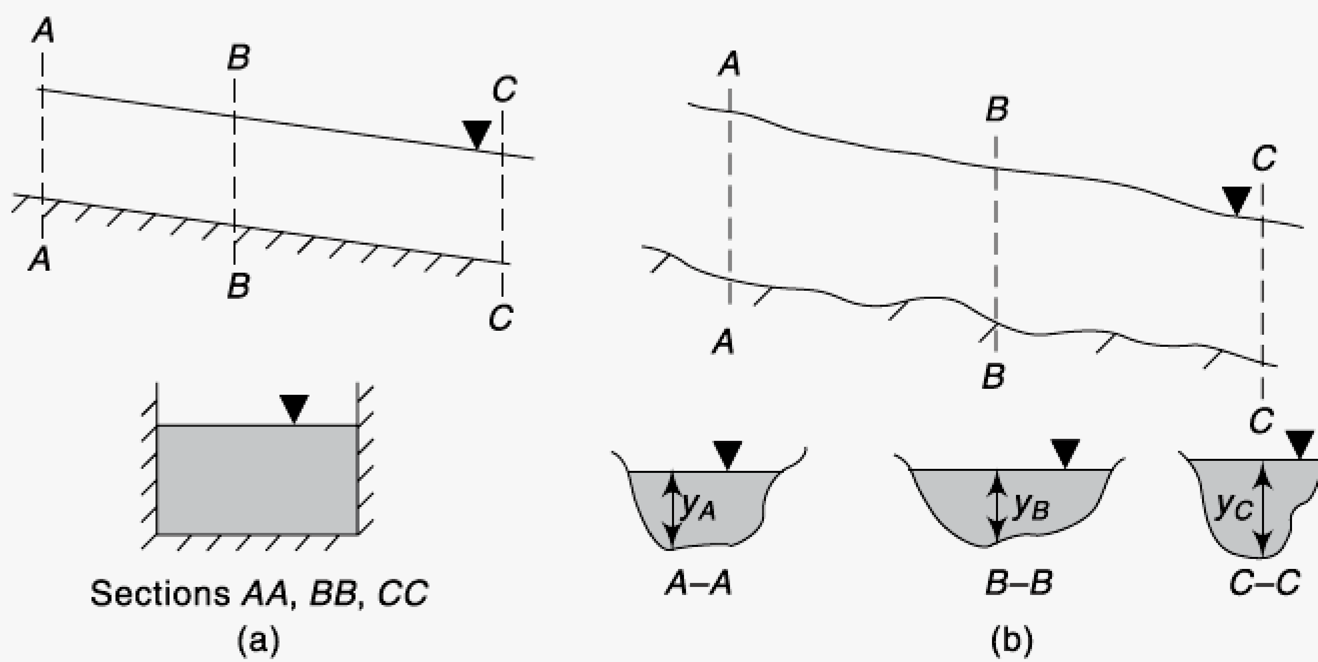



**Fig. 1.15** Prismatic and nonprismatic channels

# Classification of Open-Channel Flows

- **NATURAL AND ARTIFICIAL CHANNELS:**

A river, an estuary,[12] and a land surface during overland runoff, are all examples of natural channels while a laboratory flume, a canal, and a parking lot during overland runoff, represent artificial (or human-made) channels. Generally, natural channels would be nonprismatic while the artificial channels are likely to be prismatic. Also, natural channels typically have an irregular cross section [Fig. 1.15(b)] while artificial channels have regular (e.g., circular, trapezoidal, rectangular) cross sections [Fig. 1.15(a)].

# Classification of Open-Channel Flows

- **RIGID BOUNDARY AND MOBILE BOUNDARY CHANNELS:**

If the material on the bed and sides of a channel is loose and easily movable due to the flow of water,[13] the channel is called a *mobile boundary channel*.

Conversely, if the material is not easily movable (e.g., a metal flume, concrete lined canal), the channel is a *rigid boundary channel*.

A general mobile-boundary channel can be considered to have four degrees of freedom. For a given channel not only the depth of flow but also the bed width, longitudinal slope and planiform (or layout) of the channel may undergo changes with space and time depending on the type of flow. Mobile-boundary channels, usually treated under the topic of *sediment transport* or *sediment engineer-*

# Classification of Open-Channel Flows

## Classification Based on Flow Properties

- **TEMPORAL VARIATION- STEADY AND UNSTEADY FLOWS:**

A flow is said to be *steady* if the flow characteristics (e.g., discharge, depth, velocity) do not change with time [Fig. 1.16(a)]. If a change is observed with time, the flow is *unsteady* [Fig. 1.16(b)]. In general, it is almost impossible to have a strictly steady flow.

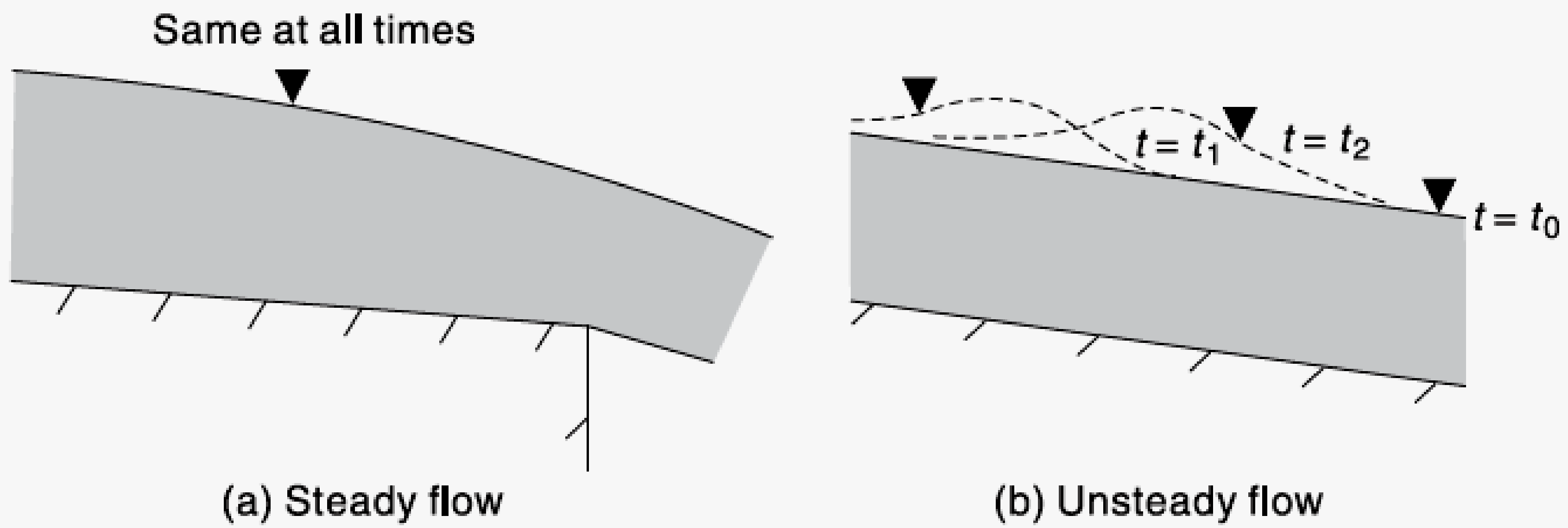



**Fig. 1.16** Steady and unsteady flows

# Classification of Open-Channel Flows

## Classification Based on Flow Properties

- **SPATIAL VARIATION- 1-D, 2-D, 3-D FLOWS:**

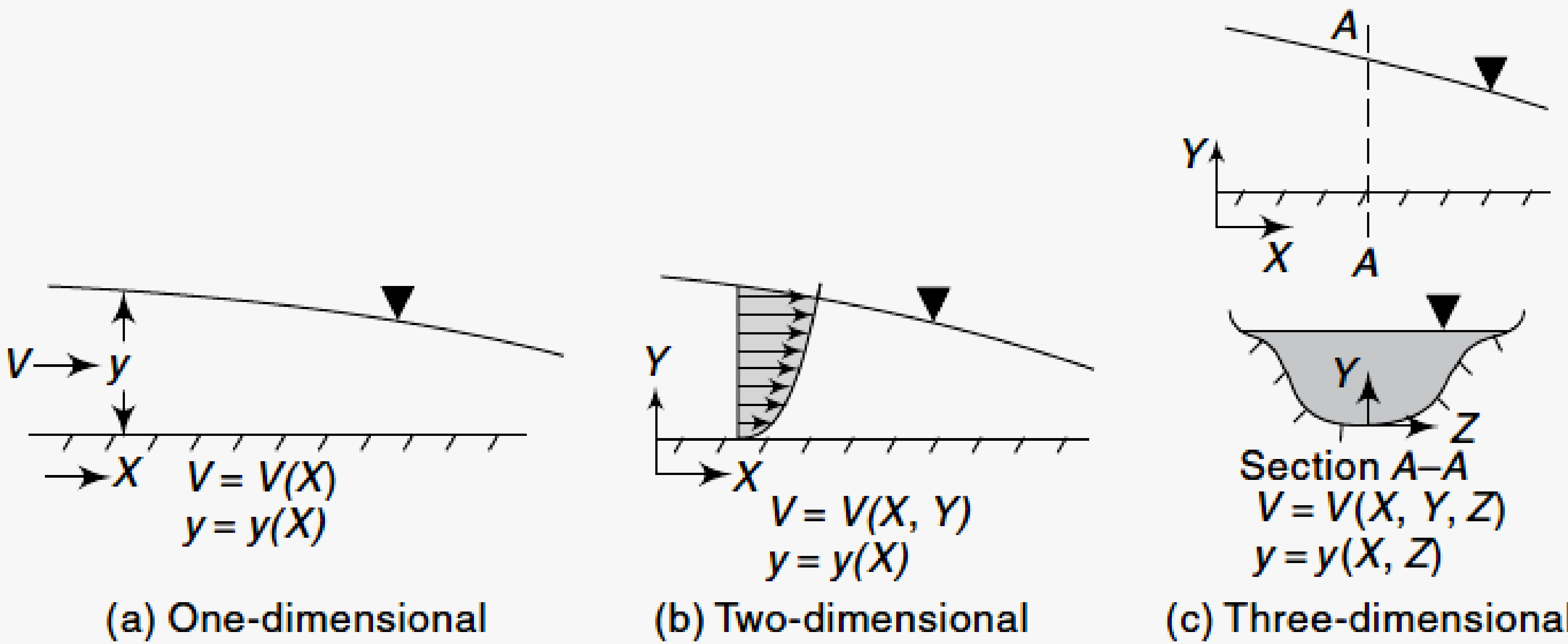



**Fig. 1.17** One-, two-, and three-dimensional flows

# Classification of Open-Channel Flows

## Classification Based on Flow Properties

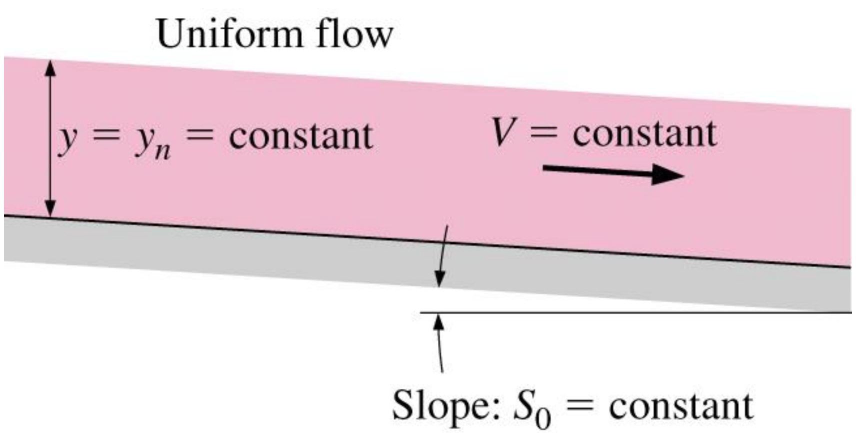
- **SPATIAL VARIATION- UNIFORM AND NONUNIFORM FLOWS:**

*Uniform flow* indicates that the flow depth, velocity, and discharge do not change in the longitudinal direction.

If the flow is not uniform, i.e., the flow depth, velocity, or discharge is varying spatially, it is known as *nonuniform (or varied) flow*.

Nonuniform flow can be further classified into gradually varied flow (GVF)[18] when the depth changes gradually, rapidly varied flow (RVF) when the depth changes significantly over a short distance, and spatially varied flow (SVF)[19] when the discharge changes due to lateral inflow or outflow.

# Classification of Open-Channel Flows




- Flow in open channels is also classified as being *uniform* or *nonuniform*, depending upon the depth *y*.
- Uniform flow (UF) encountered in long straight sections where head loss due to friction is balanced by elevation drop.
- Depth in UF is called **normal depth $y_n$**

# Classification of Open-Channel Flows

## Classification Based on Flow Properties

- **SPATIAL VARIATION- UNIFORM AND NONUNIFORM FLOWS:**

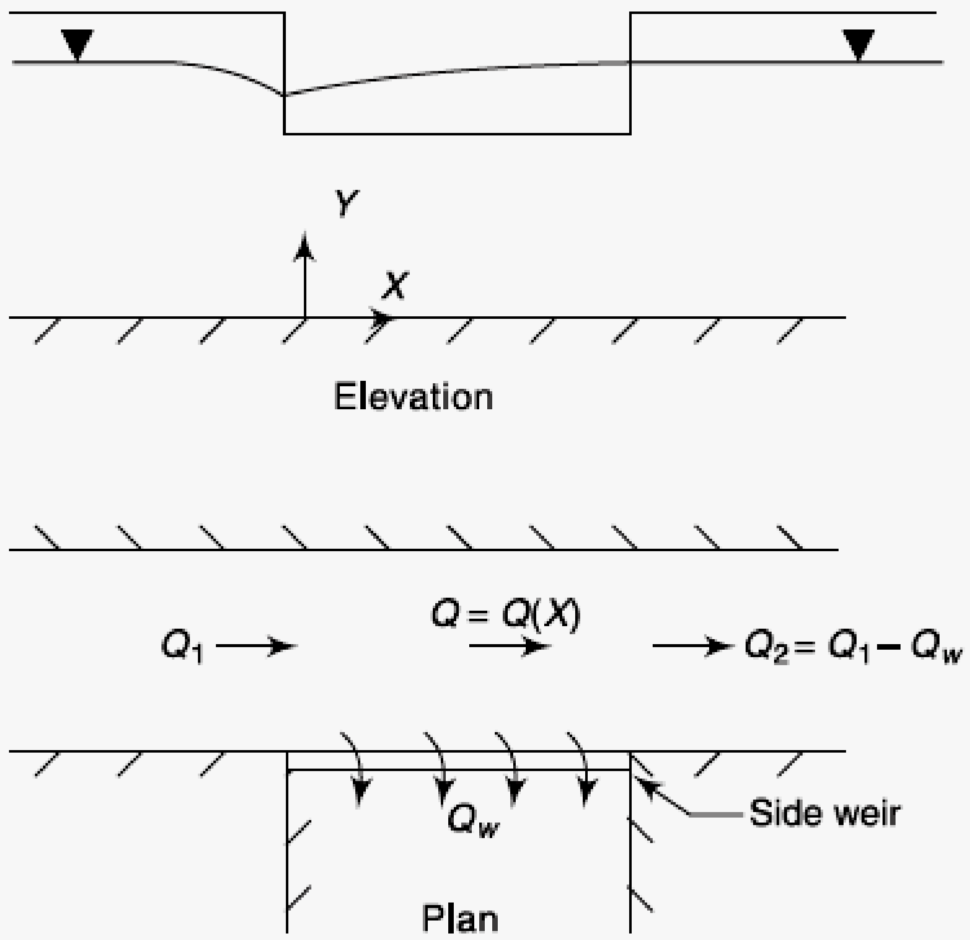



**Fig. 1.19** Spatially varied flow at a side weir

# Classification of Open-Channel Flows

## Classification Based on Flow Properties

- **EFFECT OF VISCOSITY- LAMINAR & TURBULENT FLOWS:**

*Reynolds number* $$\mathrm{Re} = \frac{\rho V L}{\mu} \quad (1.2)$$

where $\rho$ is the mass density, $V$ is the cross section average velocity, $L$ is a characteristic length, and $\mu$ is the dynamic viscosity. Generally, the pipe

HYDRAULIC RADIUS, R = A/P

FOR CIRCULAR PIPES R = $\pi * r^2 / 2 * \pi * r = r/2 = D/4$

Re FOR LAMINAR = 2000/4 = 500 FOR OCF

# Classification of Open-Channel Flows

## Classification Based on Flow Properties

- **EFFECT OF GRAVITY- CRITICAL & SUPER CRITICAL FLOWS:**

Using dimensional analysis with

inertial force $\rho V^2 L^2$ and gravitational force $\rho L^3 g$, where $V$ is the average velocity, $L$ is a characteristic length, and $g$ is the gravitational acceleration, we obtain the ratio of these forces as $V^2/gL$. Following the convention of using the first power of velocity, we define a dimensionless number (the *Froude number*) as

$$F_r = \frac{V}{\sqrt{gL}}$$

Fr = 1 CRITICAL FLOW
Fr > 1 SUPERCRITICAL FLOW
Fr< 1 SUB CRITICAL FLOW

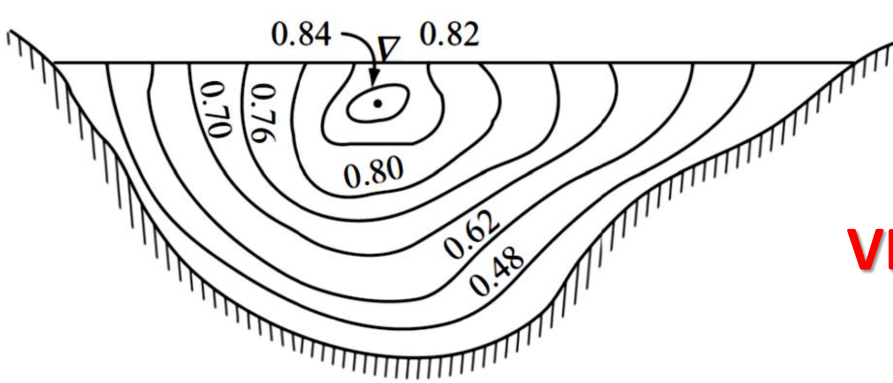
| Compressible Flow | Open-Channel Flow |
|---|---|
| $Ma = V/c$ | $Fr = V/c_0$ |
| $Ma < 1$ Subsonic<br>$Ma = 1$ Sonic<br>$Ma > 1$ Supersonic | $Fr < 1$ Subcritical<br>$Fr = 1$ Critical<br>$Fr > 1$ Supercritical |

$V$ = speed of flow
$c = \sqrt{kRT}$ = speed of sound (ideal gas)
$c_0 = \sqrt{gy}$ = speed of wave (liquid)

# VELOCITY DISTRIBUTION

- In an open channel,
  - Velocity is zero on bottom and sides of channel due to no-slip condition
  - Velocity is maximum at the midplane of the free surface
  - In most cases, velocity also varies in the streamwise direction
  - Therefore, the flow is 3D
  - Nevertheless, 1D approximation is made with good success for many practical problems.

# VELOCITY DISTRIBUTION



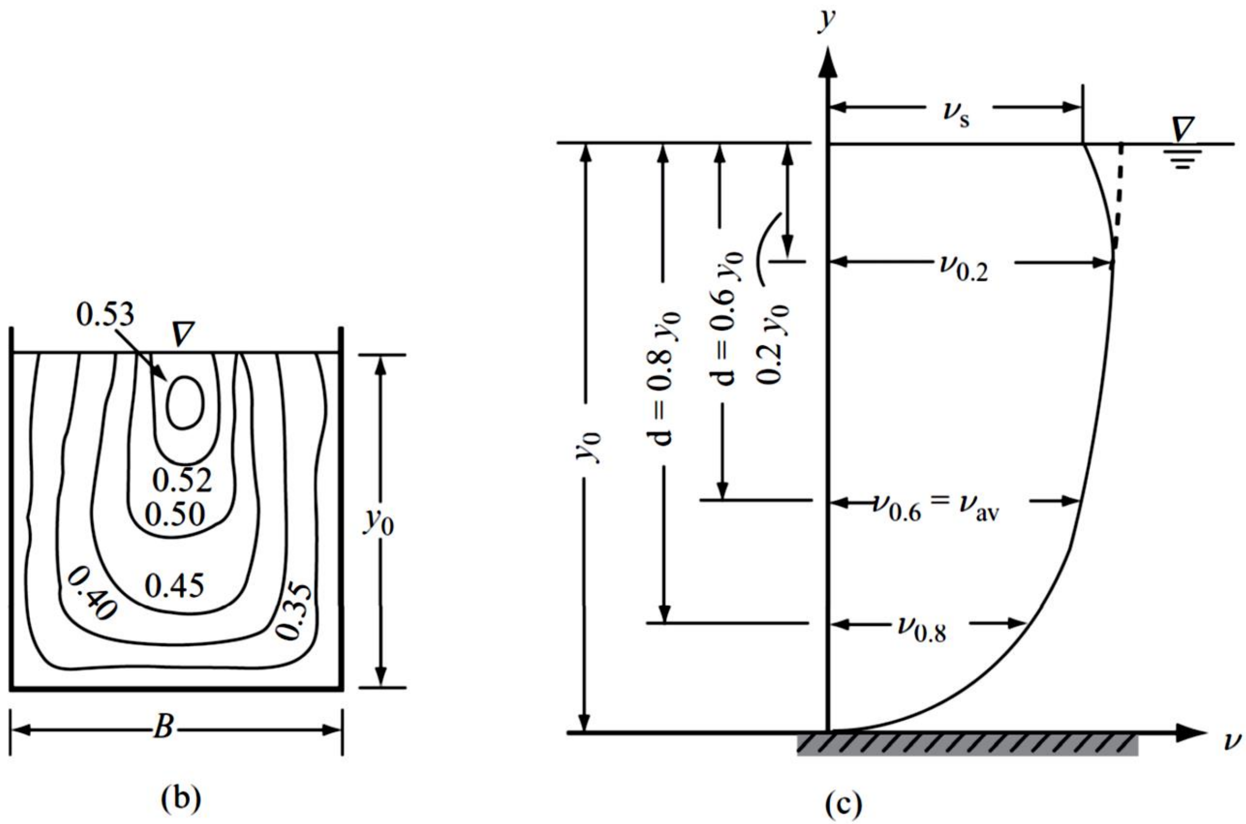




**Fig. 1.3** *Velocity distribution in open channels: (a) Natural channel (b) Rectangular channel (c) Typical velocity profile*

# VELOCITY DISTRIBUTION

Field observations in rivers and canals have shown that the average velocity at any vertical $v_{av}$, occurs at a level of 0.6 $y_0$ from the free surface, where $y_0$ = depth of flow. Further, it is found that

$$v_{av} = \frac{v_{0.2} + v_{0.8}}{2} \quad (1.1)$$

in which $v_{0.2}$ = velocity at a depth of 0.2 $y_0$ from the free surface, and $v_{0.8}$ = velocity at a depth of 0.8 $y_0$ from the free surface. This property of the velocity distribution is commonly used in stream-gauging practice to determine the discharge using the area-velocity method. The surface velocity $v_s$ is related to the average velocity $v_{av}$ as

$$v_{av} = kv_s \quad (1.2)$$

where, $k$ = a coefficient with a value between 0.8 and 0.95. The proper value of $k$ depends on the channel section and has to be determined by field calibrations. Knowing $k$, one can estimate the average velocity in an open channel by using floats and other surface velocity measuring devices.

# VELOCITY DISTRIBUTION

The presence of corners and boundaries in an open channel causes the velocity vectors of the flow to have components not only in the longitudinal and lateral direction but also in normal direction to the flow.

REVISION

MAJOR COMPONENT IS $v_x$ i.e. LONGITUDINAL VELOCITY

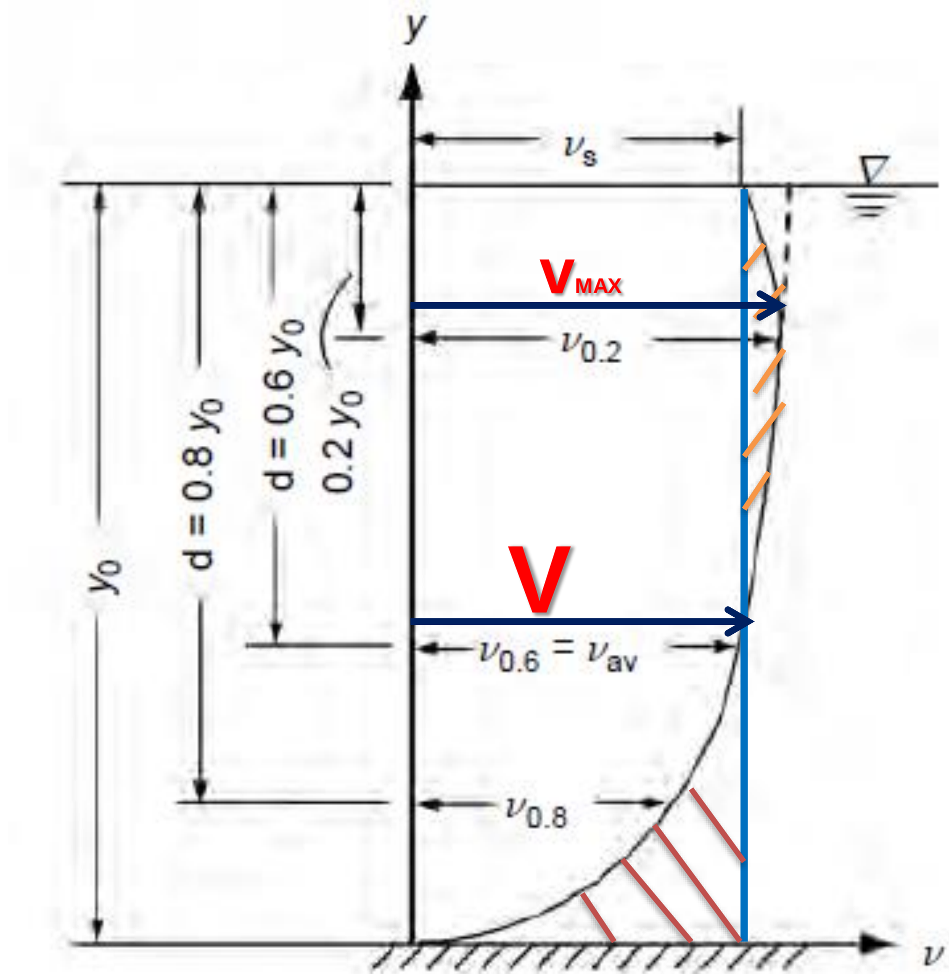
OTHER TWO COMPONENTS BEING SMALL ARE IGNORED

v IS ZERO AT SOLID BOUNDARY AND GRADUALLY INCREASES AS YOU MOVE AWAY FROM BOUNDARY

MAXIMUM VELOCTY AT A SECTION IS BELOW THE FREE SURFACE

IT IS DUE TO SECONDARY CURRENTS WHICH DEPENDS UPON ASPECT RATIO (RATIO OF DEPTH TO WIDTH)

# VELOCITY DISTRIBUTION



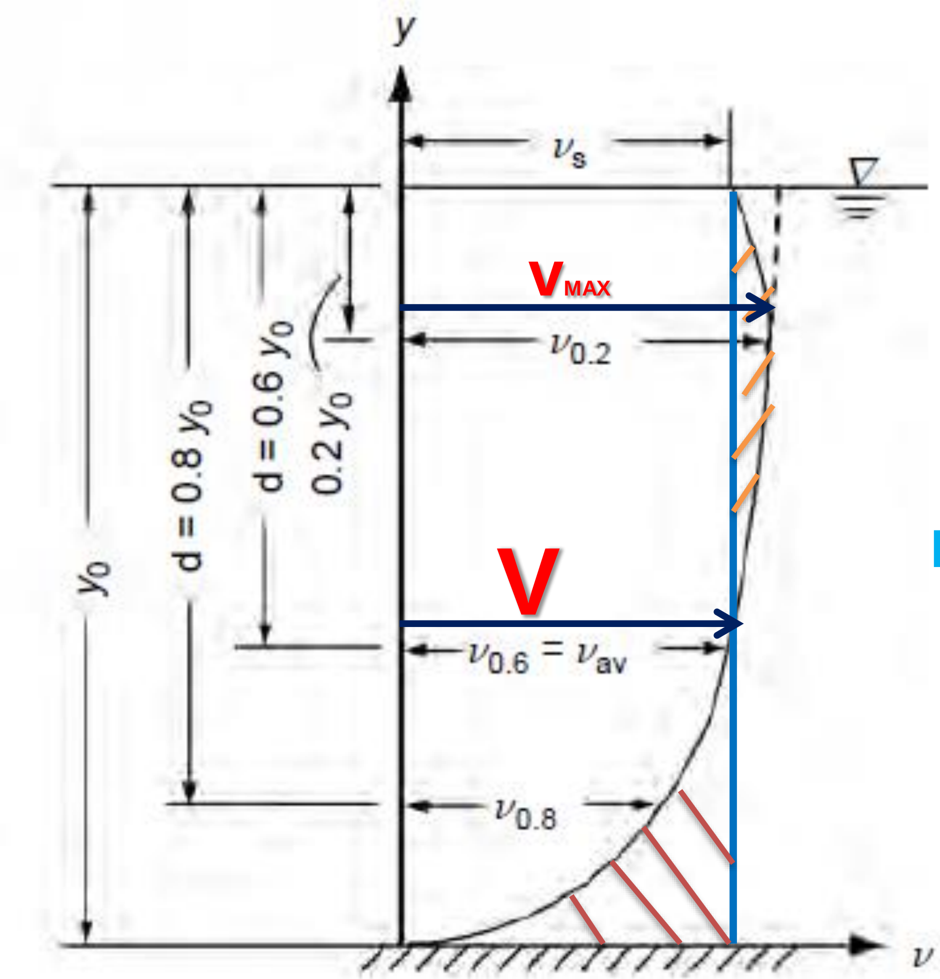

AREAS ARE EQUAL AT 0.6d

$$V = \frac{1}{A}\int_A v\,dA$$

$$Q = \int v\,dA = VA$$

# VELOCITY DISTRIBUTION

**MASS OF BOTH AREA ARE SAME BUT MOMENTUM OF UPPER AREA IS HIGHER THAN THE LOWER AREA**




**MOMENTUM = ρQv**

$$Q = B \int_0^{y0} v\, dy$$

$$M_{actual} = B\, \rho \int_0^{y0} v^2 dy$$

**M USING AVG. VELOCITY = $y_0 B\, \rho V^2$**

$$M_{actual} = \beta\, M_{average}$$

$$\beta = \frac{\int_A v^2\, dA}{V^2 A} = \frac{\Sigma v^2 \Delta A}{V^2 A}$$

β is momentum correction factor

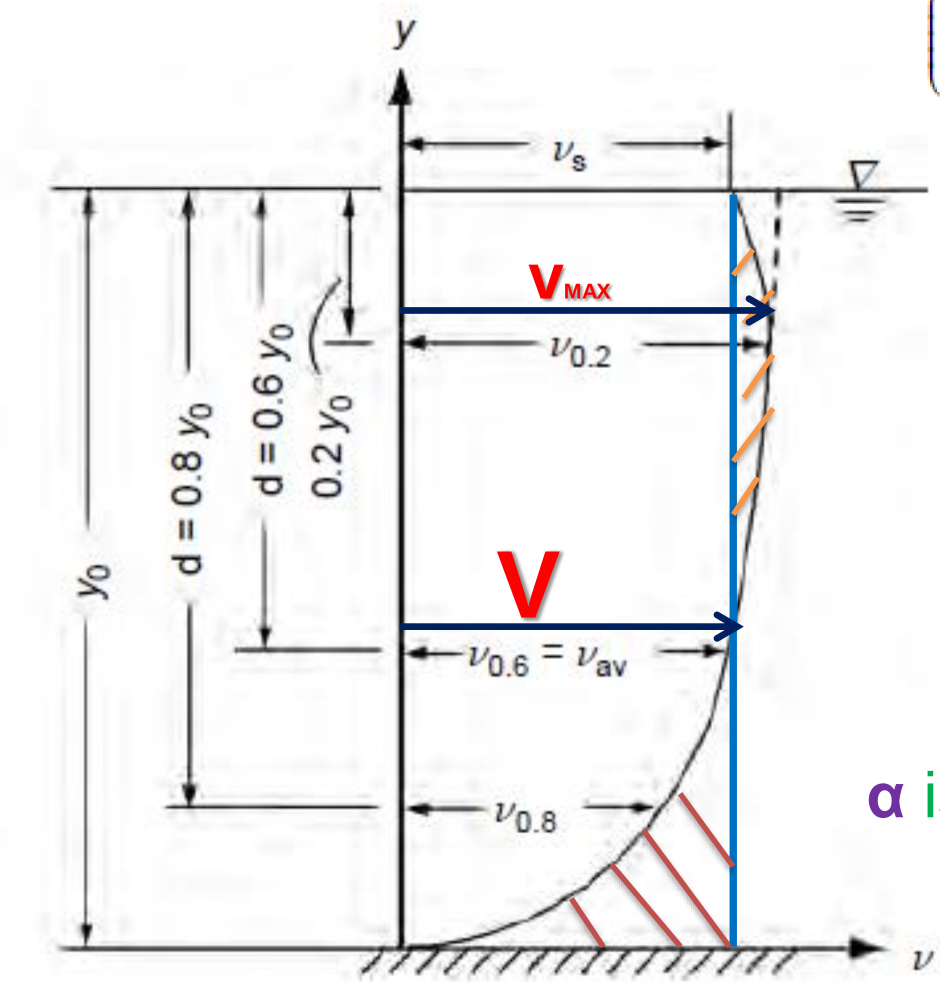
β ≥ 1.0

# VELOCITY DISTRIBUTION

## Kinetic Energy

For an elemental area $dA$, the flux of kinetic energy through it is equal to

$$\left(\frac{\text{mass}}{\text{time}}\right)\left(\frac{\text{KE}}{\text{mass}}\right) = (\rho v\, dA)\frac{v^2}{2}$$




$$KE_{actual} = \int_A \rho v^2\, dA$$

$$KE_{avg} = \alpha \frac{\rho}{2} V^3 A$$

$$KE_{actual} = \alpha\, KE_{average}$$

α is Energy Correction Factor

$$\alpha = \frac{\int v^3\, dA}{V^3 A}$$

$$\alpha \geq \beta \geq 1.0$$

# VELOCITY DISTRIBUTION

| Channels | Values of $\alpha$ | | Values of $\beta$ | |
|---|---|---|---|---|
| | Range | Average | Range | Average |
| Natural channels and torrents | 1.15 – 1.50 | 1.30 | 1.05 – 1.17 | 1.10 |
| River valleys, overflooded | 1.50 – 2.00 | 1.75 | 1.17 – 1.33 | 1.25 |

It is usual practice to assume $\alpha = \beta = 1.0$ when no other specific information about the coefficients are available.

# VELOCITY DISTRIBUTION

**Example 1.1** *The velocity distribution in a rectangular channel of width $B$ and depth of flow $y_0$ was approximated as $v = k_1\sqrt{y}$ in which $k_1$ = a constant. Calculate the average velocity for the cross section and correction coefficients $\alpha$ and $\beta$.*

*Solution* Area of cross section $A = By_0$

Average velocity
$$V = \frac{1}{By_0}\int_0^{y_0} v(B\,dy)$$

$$= \frac{1}{y_0}\int_0^{y_0} k_1\sqrt{y}\,dy = \frac{2}{3}k_1\sqrt{y_0}$$

Kinetic energy correction factor

$$\alpha = \frac{\int_0^{y_0} v^3(B\,dy)}{V^3 B\,y_0} = \frac{\int_0^{y_0} k_1^3 y^{3/2} B\,dy}{\left(\frac{2}{3}k_1\sqrt{y_0}\right)^3 B\,y_0} = 1.35$$

Momentum correction factor

$$\beta = \frac{\int_0^{y_0} v^2(B\,dy)}{V^2 B\,y_0} = \frac{\int_0^{y_0} k_1^2 y\,B\,dy}{\left(\frac{2}{3}k_1\sqrt{y_0}\right)^2 B\,y_0} = 1.125$$

# PRESSURE DISTRIBUTION

THE INTENSITY OF PRESSURE AT THE FREE SURFACE OF THE LIQUID IS EQUAL TO ATMOSPHERIC PRESSURE (GAUGE PRESSURE = 0)

EULER'S EQUATION IN ANY ARBITRARY DIRECTION *s*,

$$-\frac{\partial(p+\gamma Z)}{\partial s}=\rho a_s$$

IF *s* IS DIRECTION ALONG STREAMLINE AND *n* IS DIRECTION ACROSS IT,

$$-\frac{\partial}{\partial n}(p+\gamma Z)=\rho a_n$$

WE ARE INTERESTED IN PRESSURE DISTRIBUTION IN *n* DIRECTION

NORMAL ACCELERATION IS GIVEN BY

$$a_n=\frac{v^2}{r}$$

*v* IS VELOCITY ALONG STREAM LINE AND *r* IS RADIUS OF CURVATURE

NORMAL ACCELERATION IS = 0 WHEN *v* IS ZERO OR *r* IS $\longrightarrow \infty$

# PRESSURE DISTRIBUTION IN STILL WATER

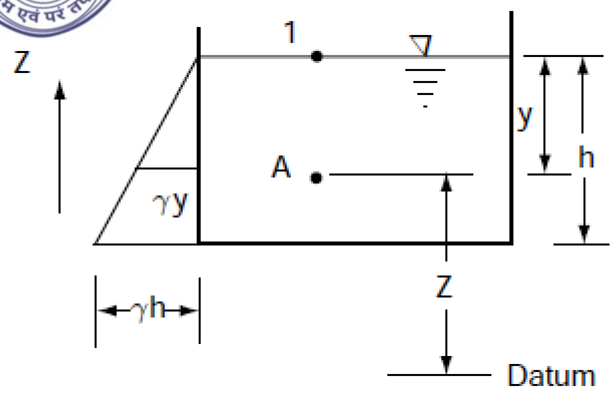



Fig. 1.4(a) *Pressure distribution in still water*

since $a_n = 0$, taking $n$ in the $Z$ direction and integrating

$$\frac{p}{\gamma} + Z = \text{constant} = C$$

$$\frac{p_A}{\gamma} = (Z_1 - Z_A) = y$$

$$p_A = \gamma y$$

# PRESSURE DISTRIBUTION IN CHANNELS WITH SMALL SLOPE

ϴ IS VERY SMALL ≈ SIN ϴ ≈ 1/1000

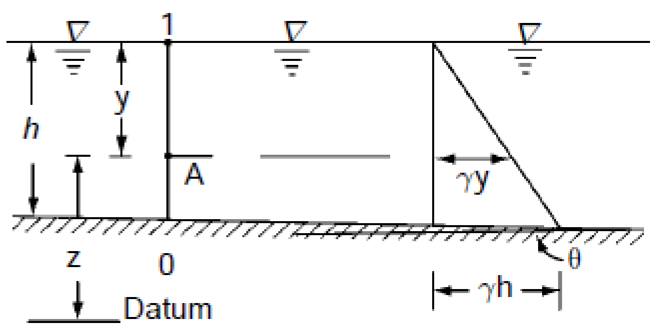



Fig. 1.4(b) *Pressure distribution in a channel with small slope*

STREAM LINES ARE PARALLEL TO CHANNEL BOTTOM SO $a_n = 0$

PRESSURE DIASTRIBUTION IN 0-1 SENTION IS HYDROSTATIC

# PRESSURE DISTRIBUTION IN CHANNELS WITH LARGE SLOPE

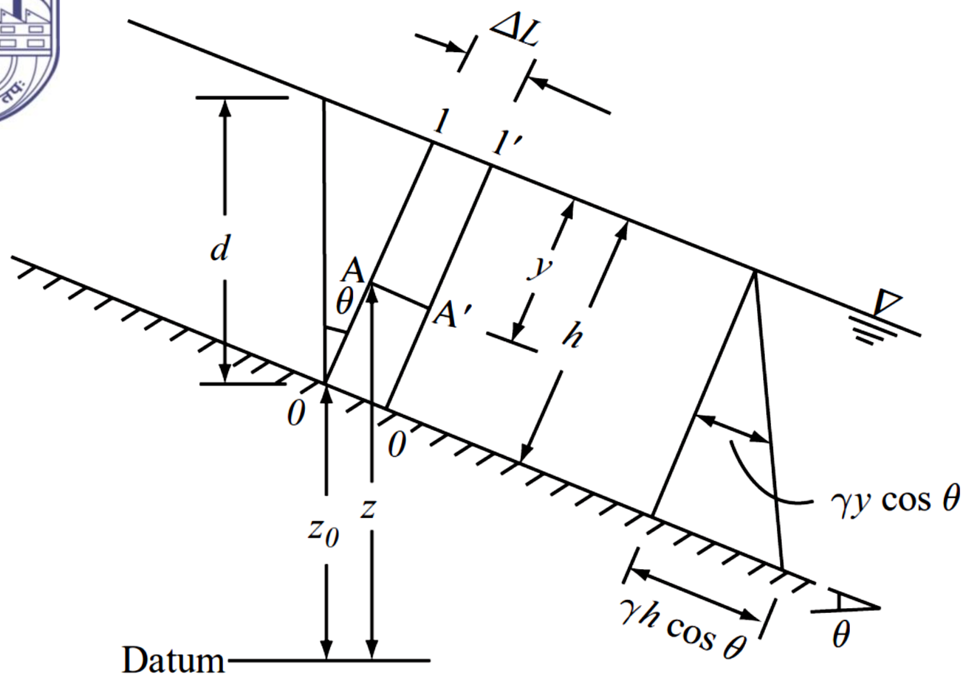



**Fig. 1.5(c)** *Pressure distribution in a channel with large slope*

weight of column $A1\ 1'A' = \gamma \Delta L y$ and acts vertically downwards.

The pressure at $AA'$ support the normal component of the column $A1\ 1'A'$. Thus $p_A \Delta L = \gamma\, y\, \Delta L \cos\theta$

$$p_A = \gamma\, y \cos\theta$$

$$p_A / \gamma = \gamma \cos\theta$$

$$\frac{p_0}{\gamma} = h\cos\theta = d\cos^2\theta$$

# PRESSURE DISTRIBUTION IN CURVILINEAR FLOWS

Figure 1.6(a) shows a curvilinear flow in a vertical plane on an upward convex surface. For simplicity consider a Section 01*A*2 in which the *r* direction and *Z* direction coincide. Replacing the *n* direction in Eq. (1.13) by (−*r*) direction,

$$\frac{\partial}{\partial r}\left[\frac{p}{\gamma}+Z\right]=\frac{a_n}{g} \quad (1.20)$$

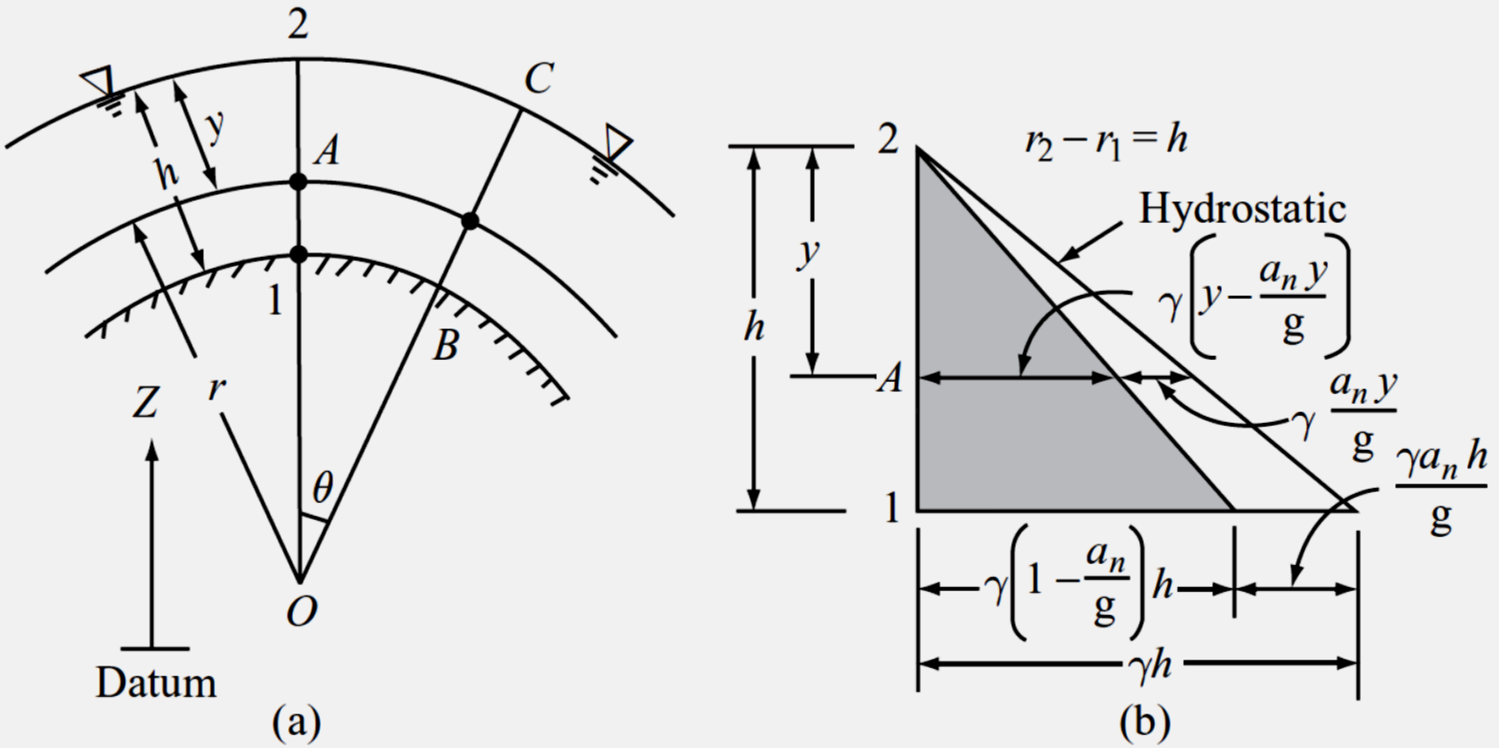



**Fig. 1.6** *Convex curvilinear flow*

giving

$$\frac{p}{\gamma} = (r_2 - r)\cos\theta - \frac{a_n}{g}(r_2 - r) \tag{1.24}$$

It may be noted that when $a_n = 0$, Eq. (1.24) is the same as Eq. (1.18a), for the flow down a steep slope.

If the curvature is convex downwards, (i.e. $r$ direction is opposite to $Z$ direction) following the argument as above, for constant $a_n$, the pressure at any point $A$ at a depth $y$ below the free surface in a vertical Section 01$A$2 [Fig. 1.7(a)] can be shown to be

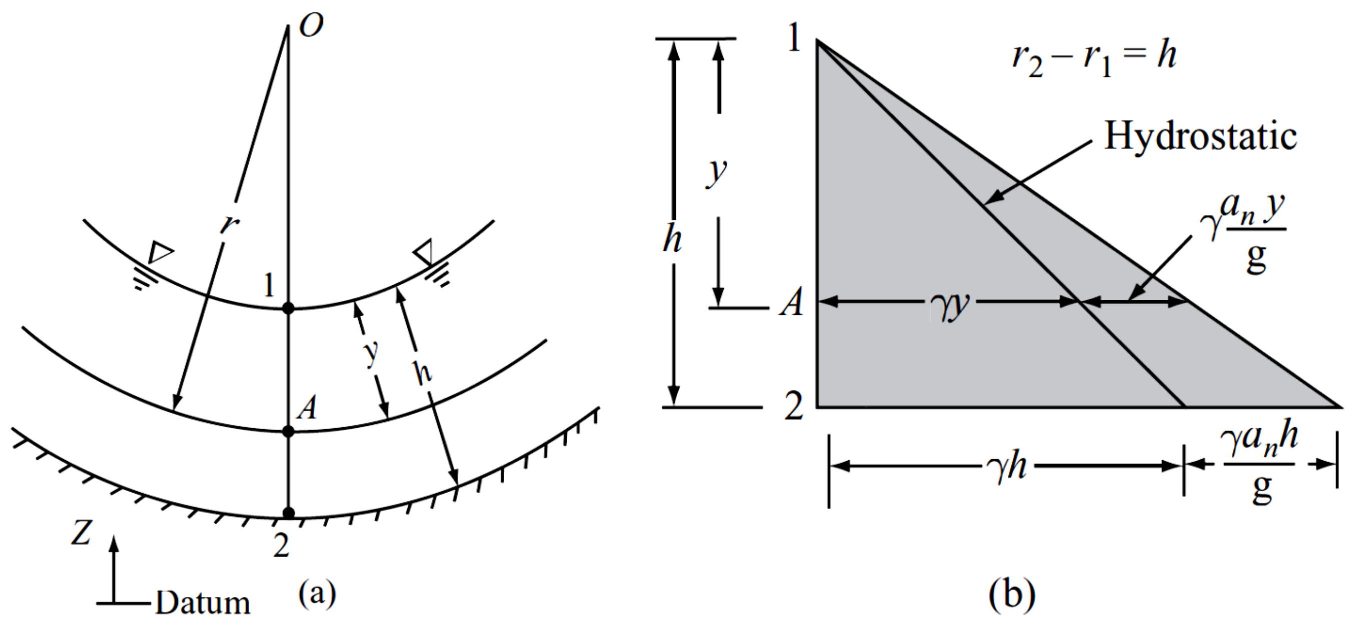



**Fig. 1.7** *Concave curvilinear flow*

$$\frac{p}{\gamma} = y + \frac{a_n}{g} y \tag{1.25}$$

The pressure distribution in a vertical section is as shown in Fig. 1.7(b).

# PRESSURE DISTRIBUTION IN CURVILINEAR FLOWS

Let us assume a simple case in which $a_n$ = constant. Then, the integration of Eq. (1.20) yields

$$\frac{p}{\gamma}+Z=\frac{a_n}{g}r+C \tag{1.21}$$

in which $C$ = constant. With the boundary condition that at point 2 which lies on the free surface, $r = r_2$ and $p/\gamma = 0$ and $Z = Z_2$,

$$\frac{p}{\gamma}=(Z_2-Z)-\frac{a_n}{g}(r_2-r) \tag{1.22}$$

Let $Z_2 - Z$ = depth below the free surface of any point $A$ in the Section $01A2 = y$. Then for point $A$,

$$(r_2-r)=y=(Z_2-Z)$$

and

$$\frac{p}{\gamma}=y-\frac{a_n}{g}y \tag{1.23}$$

Equation (1.23) shows that the pressure is less than the pressure obtained by the hydrostatic distribution [Fig. 1.6(b)].

For any normal direction $OBC$ in Fig. 1.6(a), at point $C$, $(p/\gamma)_c = 0$, $r_c = r_2$, and for any point at a radial distance $r$ from the origin $O$, by Eq. (1.22),

$$\frac{p}{\gamma}=(Z_c-Z)-\frac{a_n}{g}(r_2-r)$$

But

$$Z_c-Z=(r_2-r)\cos\theta,$$

. .

# CONTINUITY EQUATION

**STEADY FLOW** $Q = VA = V_1 A_1 = V_2 A_2 = \dots$

**UNSTEADY FLOW: SVF**

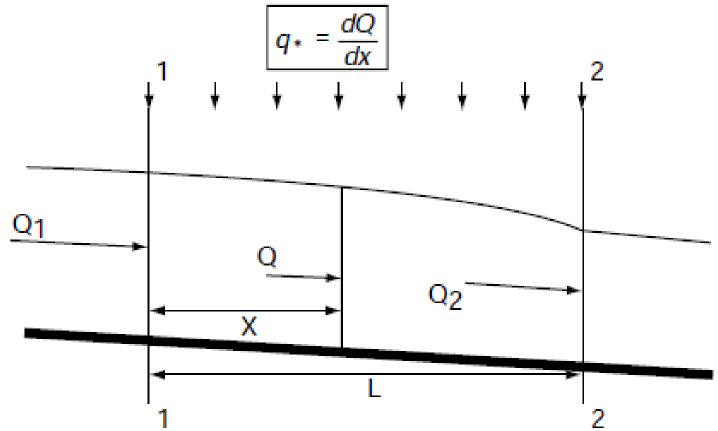



Fig. 1.11 *Spatially varied flow*

for example, an SVF with increasing discharge as in Fig. 1.11.

The rate of addition of discharge $= dQ/dx = q_*$.

The discharge at any section at a distance $x$ from Section 1 $= Q = Q_1 + \int_0^x q_* dx$

If $q_*$ = constant, $Q = Q_1 + q_* x$ and $Q_2 = Q_1 + q_* L$

# UNSTEADY FLOW

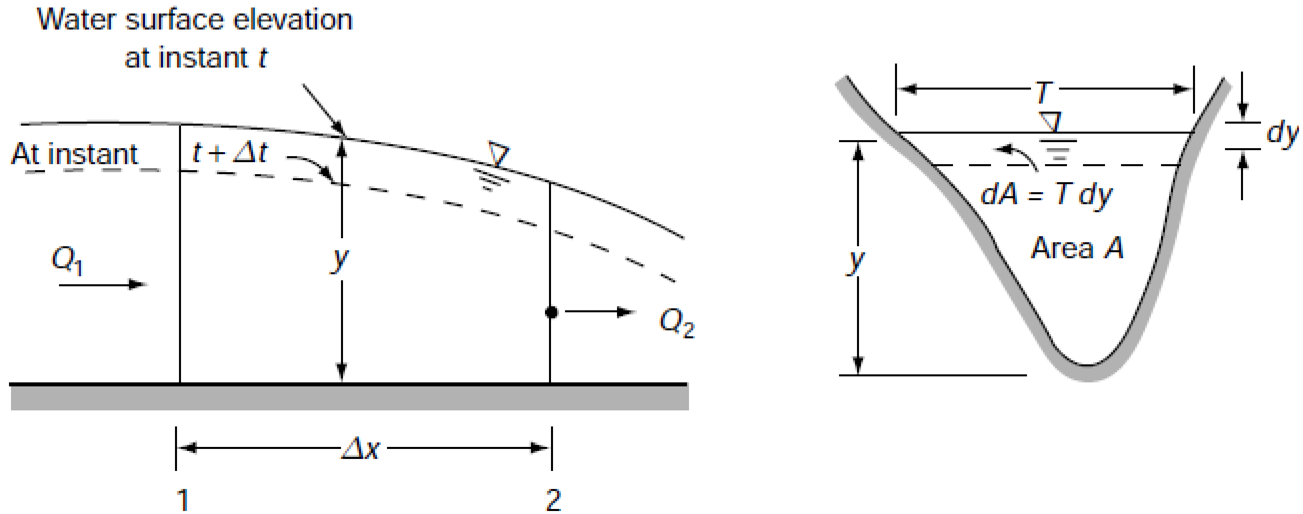



Fig. 1.12 *Definition sketch of unsteady flow*

$$Q_2 - Q_1 = \frac{\partial Q}{\partial x}\Delta x$$

Inflow in time Δt = QΔt

Net flow : $$Q\Delta t - \left(Q + \frac{\partial Q}{\partial x}\Delta x\right)\Delta t + q_x \Delta x \Delta t$$

Change in storage: $$\frac{\partial}{\partial t}(A\,\Delta x)\,\Delta t, \qquad q_x - \frac{\partial Q}{\partial x} = \frac{\partial A}{\partial t}$$

$$\boxed{\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = \pm\, q_x}$$

Equation of continuity for unsteady flow

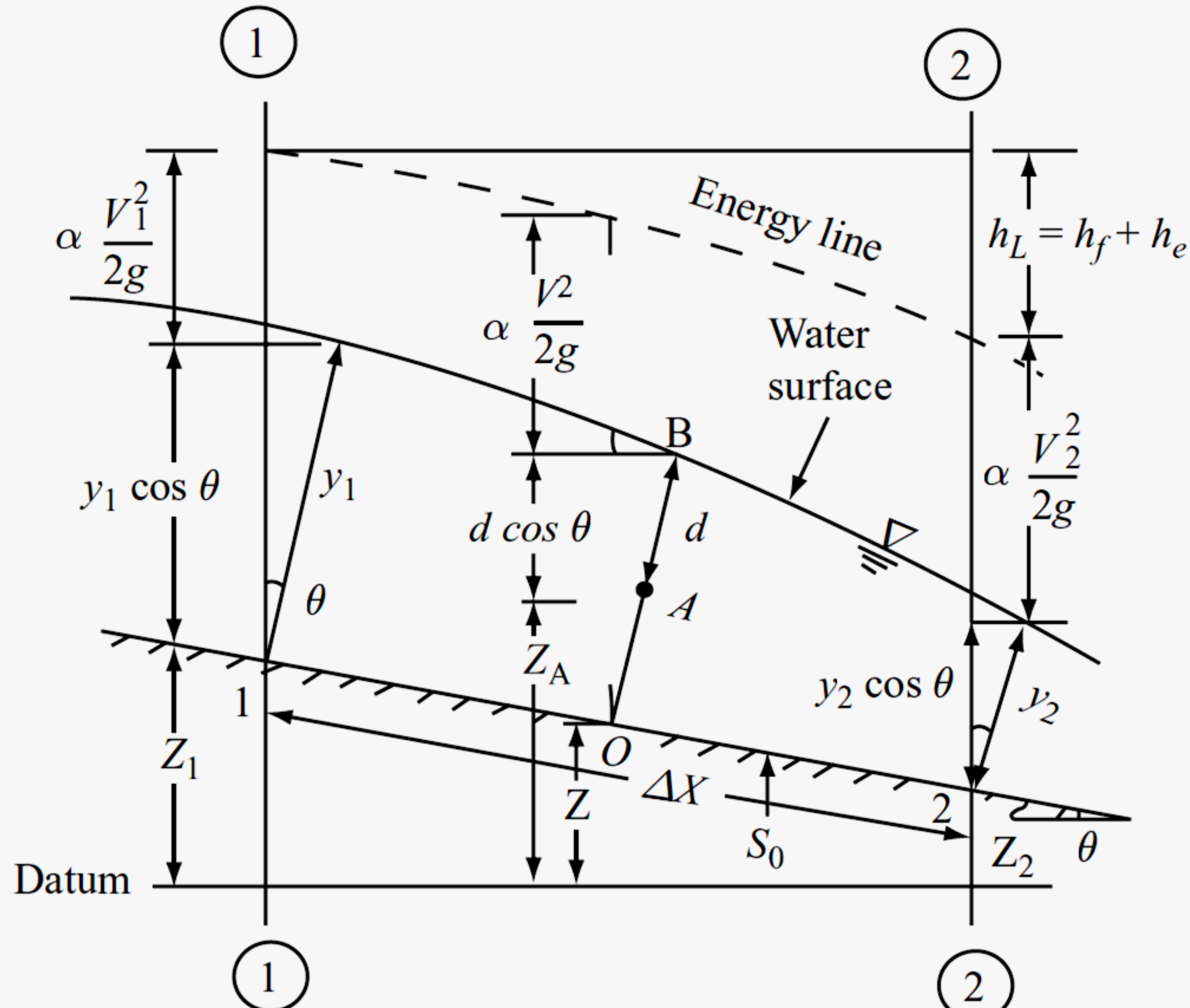



**Fig.1.14** *Definition sketch for the energy equation*

# STEADY FLOW MOMENTUM EQUATION

Momentum is a vector quantity. The momentum equation commonly used in most of the open-channel flow problems is the *linear-momentum equation*. This equation states that the algebraic sum of all external forces, acting in a given direction on a fluid mass equals the time rate of change of linear-momentum of the fluid mass in the direction. In a steady flow, the rate of change of momentum in a given direction will be equal to the net flux of momentum in that direction.

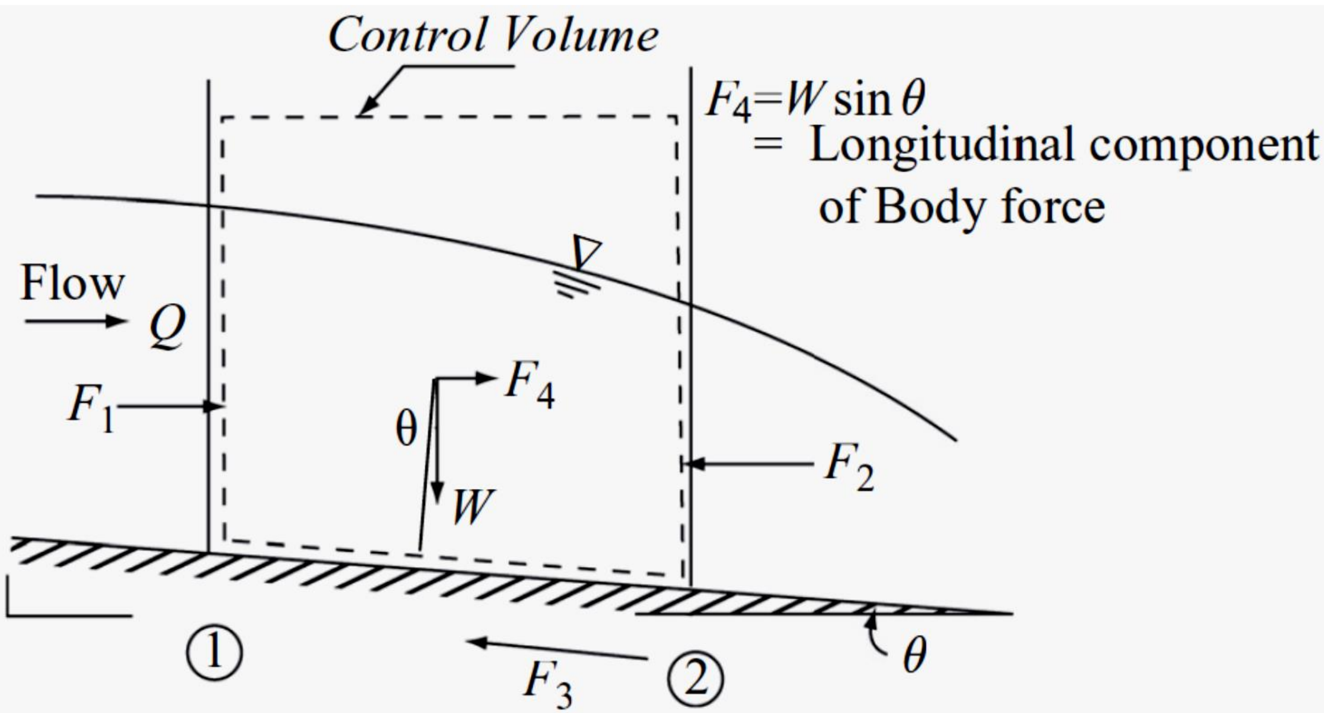



**Fig.1.17** *Definition sketch for the momentum equation*

Figure 1.17 shows a *control volume* (a volume fixed in space) bounded by sections 1 and 2, the boundary and a surface lying above the free surface. The various forces acting on the control volume in the longitudinal direction are as follows:

(i) Pressure forces acting on the control surfaces, $F_1$ and $F_2$.

(ii) Tangential force on the bed, $F_3$,

(iii) Body force, i.e., the component of the weight of the fluid in the longitudinal direction, $F_4$.

By the linear-momentum equation in the longitudinal direction for a steady-flow discharge of $Q$,

$$\Sigma F_1 = F_1 - F_2 - F_3 + F_4 = M_2 - M_1 \tag{1.45}$$

in which $M_1 = \beta_1 \rho Q V_1$ = momentum flux entering the control volume, $M_2 = \beta_2 \rho Q V_2$ = momentum flux leaving the control volume.

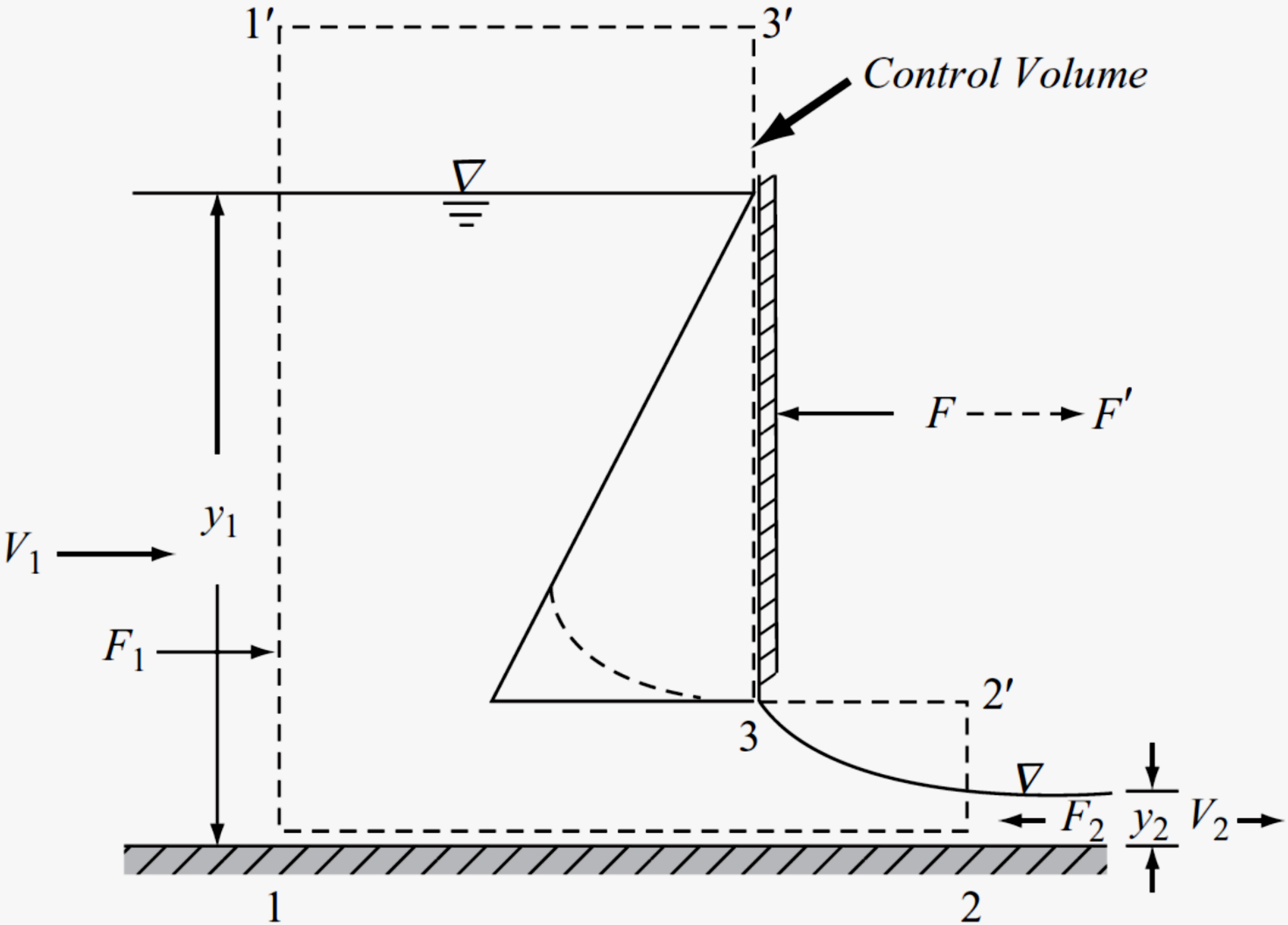




**Fig. 1.18** *Forces in a sluice gate glow—Example 1.9*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