Subject Code: NCE 202

Subject Name: Hydraulics & Hydraulic Machines

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UNIT -3

GRADUALLY VARIED FLOW

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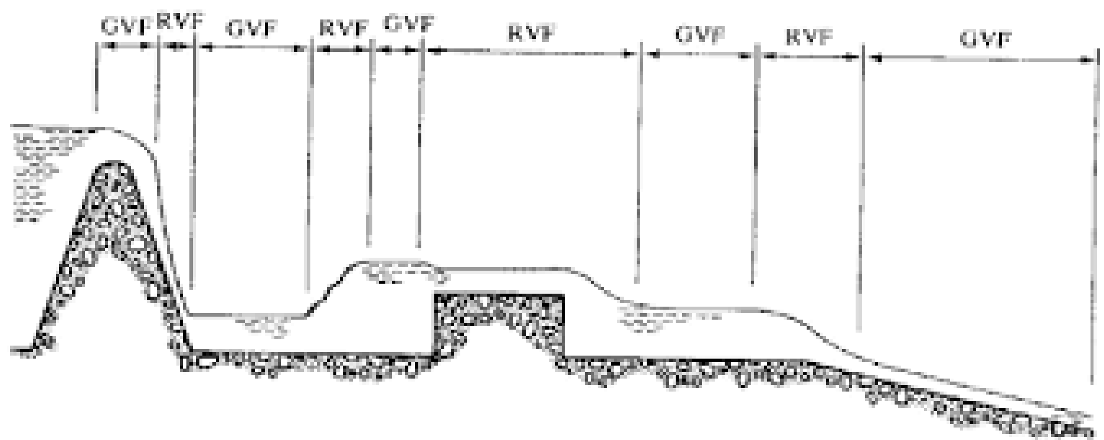
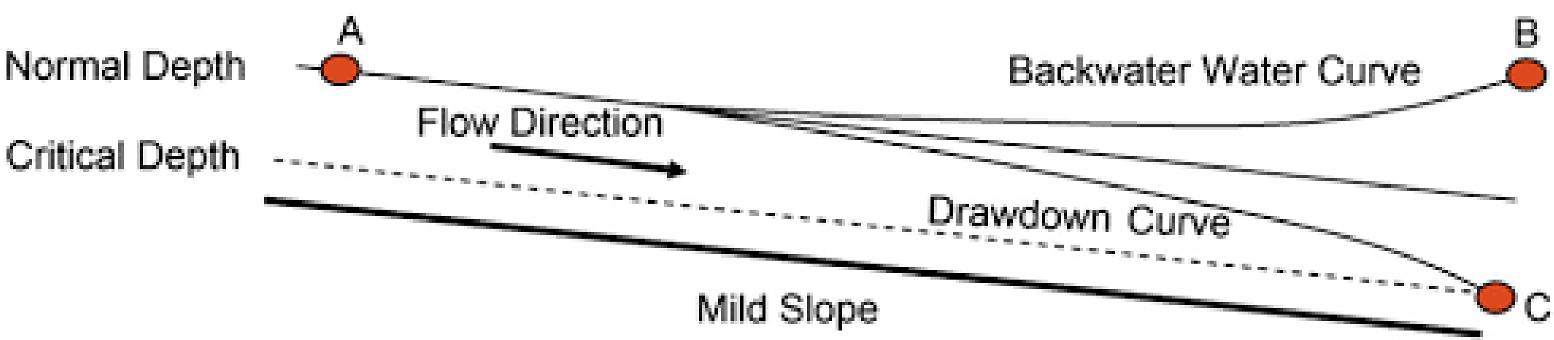
INTRODUCTION

A Steady non uniform flow in a prismatic channel with gradual changes in water surface elevation





INTRODUCTION

TWO BASIC ASSUMPTIONS:

1. The pressure distribution at any section is assumed to be hydrostatic

Gradual change in surface, acceleration is = 0, so pressure is hydrostatic

2. The energy slope $S_f = \frac{n^2 V^2}{R^{4/3}}$

Same as Manning's equation

INTRODUCTION

Some additional assumptions :

1. The slope of the channel is small, so that

    a. Depth of flow is same
    b. Pressure correction factor $\cos\theta \approx 1$
    c. No air entrainment occurs (mixing with air)

2. The channel is prismatic

    Channel has constant alignment and shape

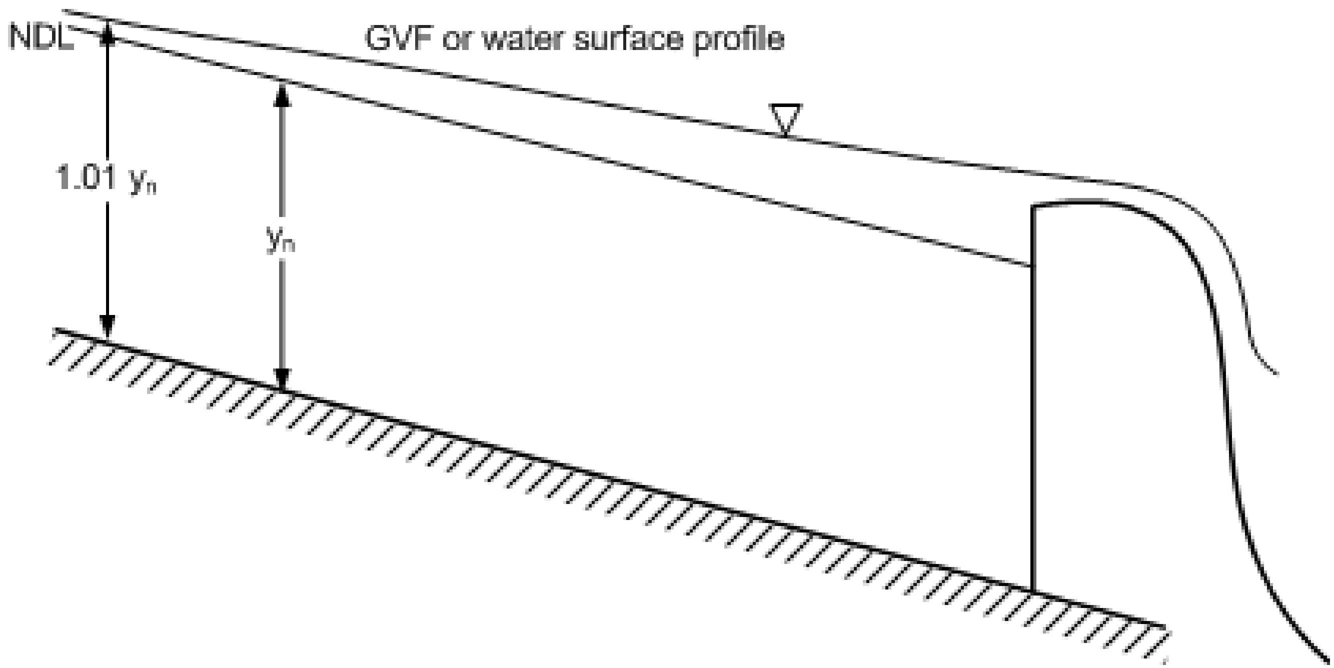
3. Velocity distribution in channel section is fixed

    Velocity distribution coefficients are constant

4. Conveyance K and Section Factor Z are exponential functions of depth of flow

5. The roughness coefficient is independent of the depth of flow and constant throughout the channel

INTRODUCTION



DIFFERENTIAL EQUATION OF GVF

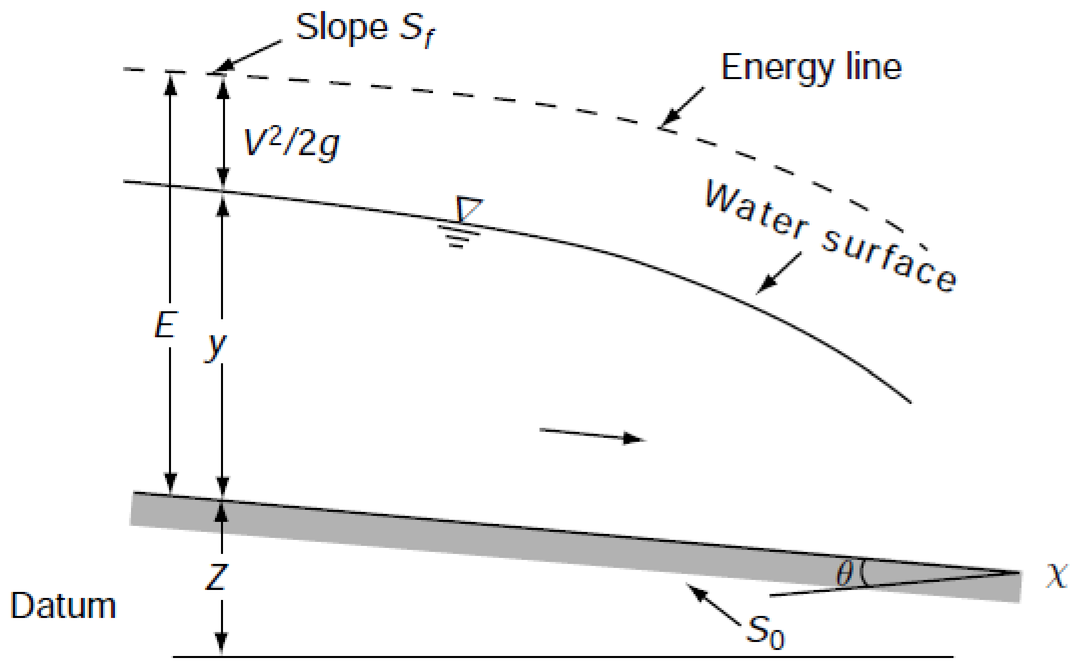


Fig. 4.1 *Schematic sketch of GVF*

Consider the total energy H of a gradually varied flow in a channel of small slope and $\alpha = 1.0$ as

$$H = Z + E = Z + y + \frac{V^2}{2g}$$

DIFFERENTIAL EQUATION OF GVF

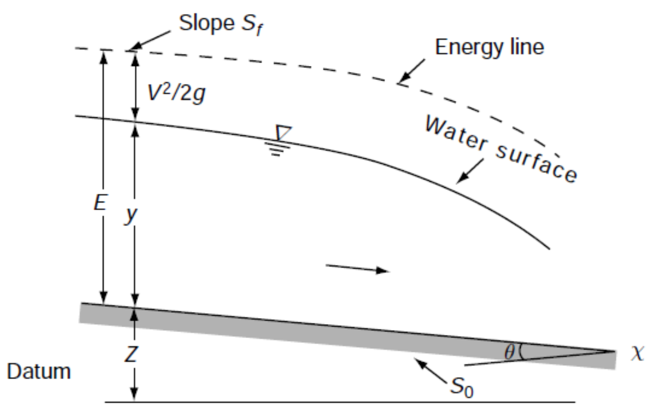


Fig. 4.1 *Schematic sketch of GVF*

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dE}{dx}$$

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$

$\frac{dH}{dx}$ represents the energy slope.

$$\frac{dH}{dx} = -S_f$$

$\frac{dZ}{dx}$ denotes the bottom slope.

$$\frac{dZ}{dx} = -S_0$$

$\frac{dy}{dx}$ represents the water surface slope relative to the bottom of the channel.

DIFFERENTIAL EQUATION OF GVF

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right)=\frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right)\frac{dy}{dx} \qquad =-\frac{Q^2}{gA^3}\frac{dA}{dy}\frac{dy}{dx}$$

Since $dA/dy = T$,

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right)=-\frac{Q^2T}{gA^3}\frac{dy}{dx}$$

$$-S_f=-S_0+\frac{dy}{dx}-\left(\frac{Q^2T}{gA^3}\right)\frac{dy}{dx}$$

Re-arranging

$$\frac{dy}{dx}=\frac{S_0-S_f}{1-\dfrac{Q^2T}{gA^3}}$$

This forms the basic differential equation of GVF and is also known as the *dynamic equation* of GVF.

DIFFERENTIAL EQUATION OF GVF

If a value of the kinetic-energy correction factor α greater than unity is to be used, Eq. 4.8 would then read as

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \dfrac{\alpha Q^2 T}{g A^3}}$$

Other Forms of Eq.

(a) If K = conveyance at any depth y and K_0 = conveyance corresponding to the normal depth y_0, then

$$K = Q/\sqrt{S_f} \quad \text{(By assumption 2 of GVF)}$$

and $K_0 = Q/\sqrt{S_0}$ (Uniform flow) $\quad S_f/S_0 = K_0^2/K^2$

Similarly, if Z = section factor at depth y and Z_c = section factor at the critical depth y_c,

$$Z^2 = A^3/T \qquad Z_c^2 = \frac{A_c^3}{T_c} = \frac{Q^2}{g}$$

DIFFERENTIAL EQUATION OF GVF

Hence, $$\frac{Q^2 T}{gA^3} = \frac{Z_c^2}{Z^2}$$

$$\frac{dy}{dx} = S_0 \frac{1 - \frac{S_f}{S_0}}{1 - \frac{Q^2 T}{g\mathrm{A}^3}} \qquad = S_0 \frac{1 - \left(\frac{K_0}{K}\right)^2}{1 - \left(\frac{Z_c}{Z}\right)^2}$$

(b) If Q_n represents the normal discharge at a depth y and Q_c denotes the critical discharge at the same depth y,

$$Q_n = K\sqrt{S_0} \qquad Q_c = Z\sqrt{g}$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{gA^3}} \qquad \frac{dy}{dx} = S_0 \frac{1 - (Q/Q_n)^2}{1 - (Q/Q_c)^2}$$

DIFFERENTIAL EQUATION OF GVF

(c) Another form of Eq. can be written as $$\frac{dE}{dx} = S_0 - S_f$$

$$\frac{Q^2 T}{g A^3} = \frac{V^2 T}{g A} = \frac{V^2}{g \frac{A}{T}} = Fr^2$$

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$ Governing equation for gradually varied flow

- Gives change of water depth with distance along channel
- Note
  - S_o and S_f are positive when sloping down in direction of flow
  - y is measured from channel bottom
  - dy/dx =0 means water depth is constant

y_n is when $S_o = S_f$

DIFFERENTIAL EQUATION OF GVF

$$\frac{dy}{dx} = 0$$

Then $S_0 = S_f$ and slope of water surface S_w = Bottom slope $S_{0,}$
Uniform Flow

$$\frac{dy}{dx} > 0$$

$S_w < S_{0,}$ Backwater Curve

$$\frac{dy}{dx} < 0$$

$S_w > S_{0,}$ Drawdown Curve

$$\frac{dy}{dx} = S_0 \frac{1 - \frac{S_f}{S_0}}{1 - \frac{Q^2 T}{gA^3}} = S_0 \frac{1 - \left(\frac{K_0}{K}\right)^2}{1 - \left(\frac{Z_c}{Z}\right)^2} = S_0 \frac{1 - \left(\frac{y_0}{y}\right)^N}{1 - \left(\frac{y_c}{y}\right)^M}$$

DIFFERENTIAL EQUATION OF GVF

$$\frac{dy}{dx} = S_0 \frac{1 - \left(\frac{y_0}{y}\right)^N}{1 - \left(\frac{y_c}{y}\right)^M}$$

$\frac{dy}{dx} > 0$

- If $y > y_0$ and $y > y_c$
  or
- $y < y_0$ and $y < y_c$

DIFFERENTIAL EQUATION OF GVF

$$\frac{dy}{dx} = S_0 \frac{1 - \left(\frac{y_0}{y}\right)^N}{1 - \left(\frac{y_c}{y}\right)^M}$$

$\frac{dy}{dx} < 0$

- If $y_c > y > y_0$ or
- $y_0 > y > y_c$

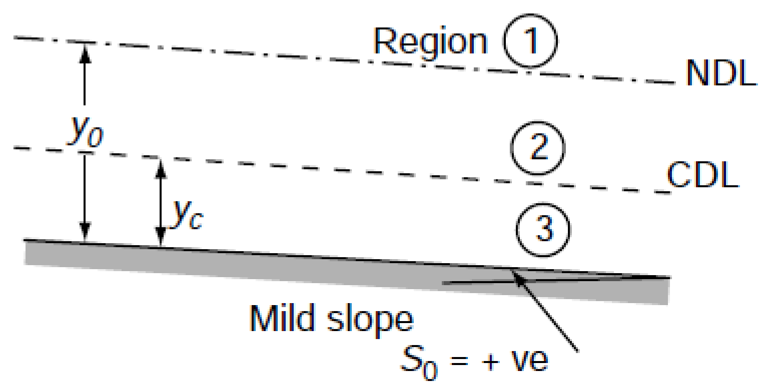
CLASSIFICATION OF GVF PROFILES

In a given channel, y_0 and y_c are two fixed depths if Q, n and S_0 are fixed. there are three possible relations between y_0 and y_c as

$$\text{(i) } y_0 > y_c, \quad \text{(ii) } y_0 < y_c \text{ and } \quad \text{(iii) } y_0 = y_c.$$

there are two cases where y_0 does not exist, when (a) the channel bed is horizontal, ($S_0 = 0$) (b) when the channel has an adverse slope, (S_0 is –ve)

three regions



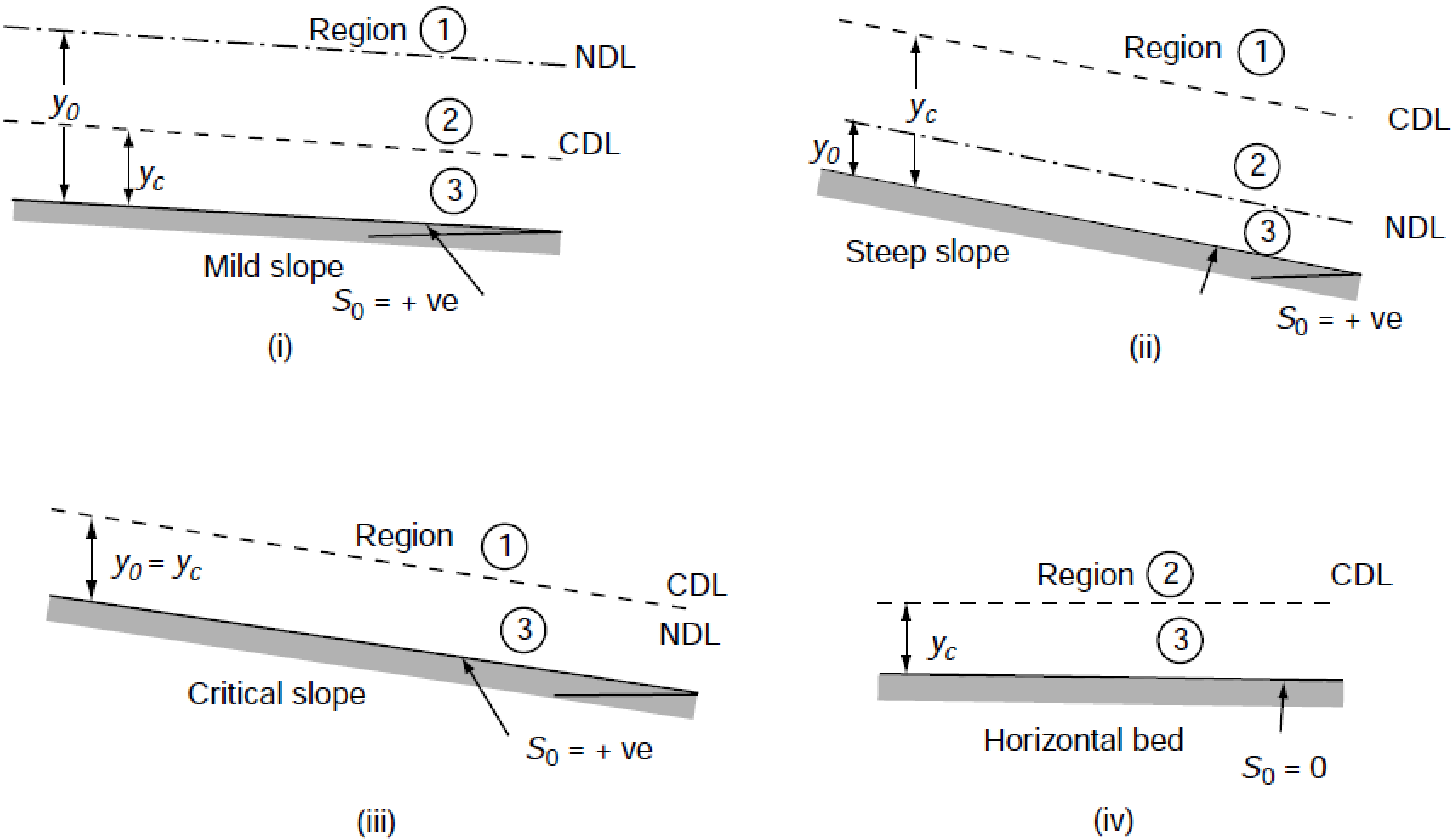
Region 1: Space above the top most line
Region 2: Space between top line and the next lower line
Region 3: Space between the second line and the bed

CLASSIFICATION OF GVF PROFILES

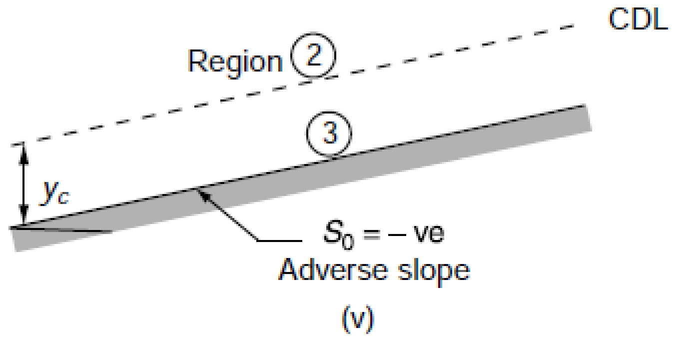
- **M**ild slope ($y_n > y_c$)
  - in a long channel subcritical flow will occur
- **S**teep slope ($y_n < y_c$)
  - in a long channel supercritical flow will occur
- **C**ritical slope ($y_n = y_c$)
  - in a long channel unstable flow will occur
- **H**orizontal slope ($S_o = 0$)
  - y_n undefined
- **A**dverse slope ($S_o < 0$)
  - y_n undefined

Note: These slopes are f(Q)!

WATER SURFACE PROFILES OF GVF



WATER SURFACE PROFILES OF GVF



C2 H1 A1 Non existent profiles

WATER SURFACE PROFILES OF GVF

Table 4.2 *Types of Gradually Varied Flow (GVF) Profiles*

Channel	Region	Condition	Type
Mild slope **(M)**	1	$y > y_0 > y_c$	M_1
	2	$y_0 > y > y_c$	M_2
	3	$y_0 > y_c > y$	M_3
Steep slope **(S)**	1	$y > y_c > y_0$	S_1
	2	$y_c > y > y_0$	S_2
	3	$y_c > y_0 > y$	S_3
Critical slope **(C)**	1	$y > y_0 = y_c$	C_1
	3	$y < y_0 = y_c$	C_3
Horizontal bed **(H)**	2	$y > y_c$	H_2
	3	$y < y_c$	H_3
Adverse slope **(A)**	2	$y > y_c$	A_2
	3	$y < y_c$	A_3

WATER SURFACE PROFILES OF GVF

$$\frac{dy}{dx} = S_0 \frac{1 - \left(\frac{y_0}{y}\right)^N}{1 - \left(\frac{y_c}{y}\right)^M}$$

NDL = *Normal depth line*
CDL = *Critical depth line*

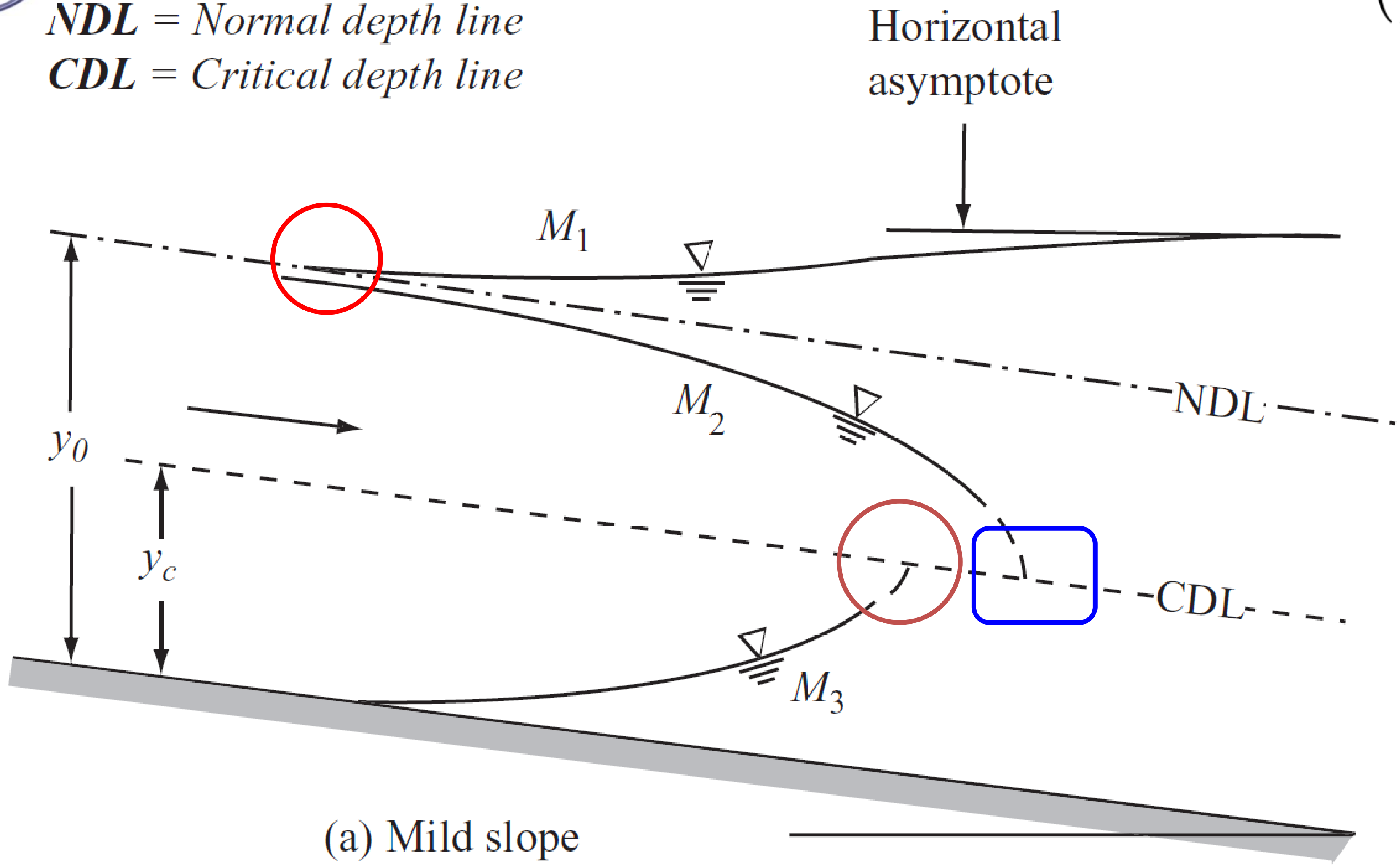
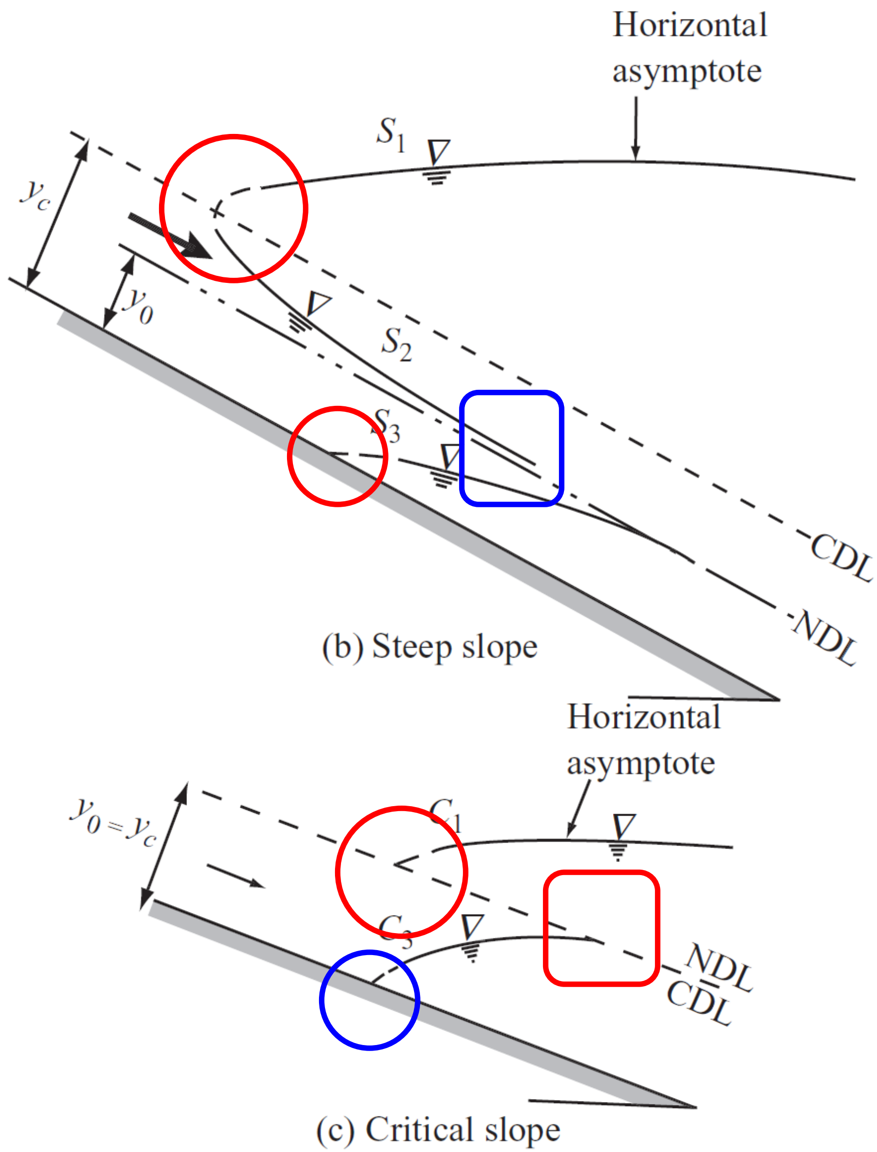


Fig. 4.3 *Various GVF Profiles (Contd)*

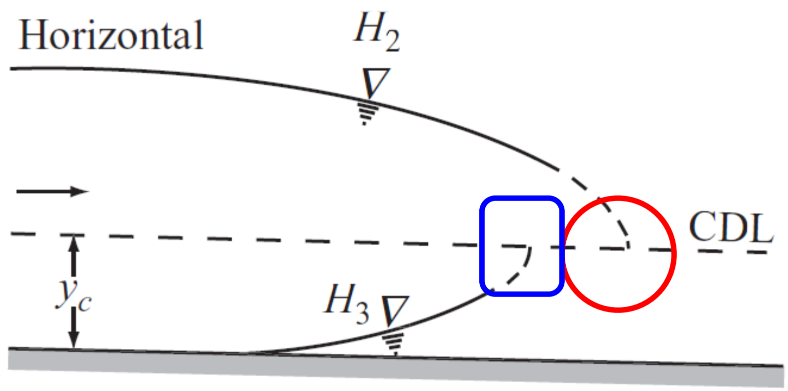
WATER SURFACE PROFILES OF GVF



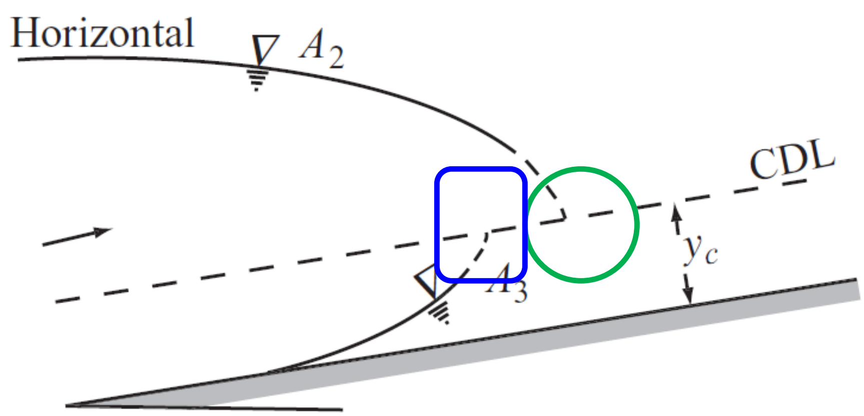
(b) Steep slope

(c) Critical slope

WATER SURFACE PROFILES OF GVF



(d) Horizontal bed



(e) Adverse slope

Fig. 4.3 *Various GVF profiles*

WATER SURFACE PROFILES OF GVF

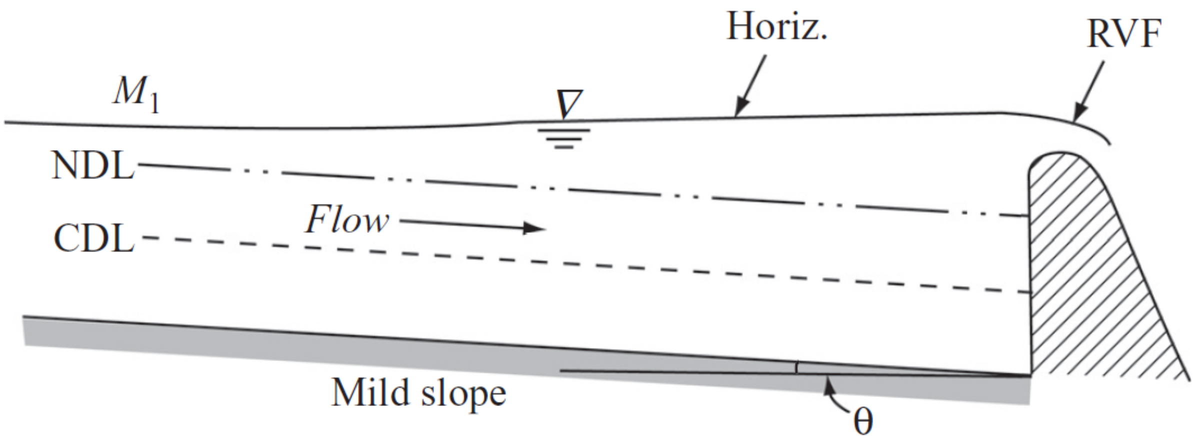


Fig. 4.4 (a) *M_1 profile*

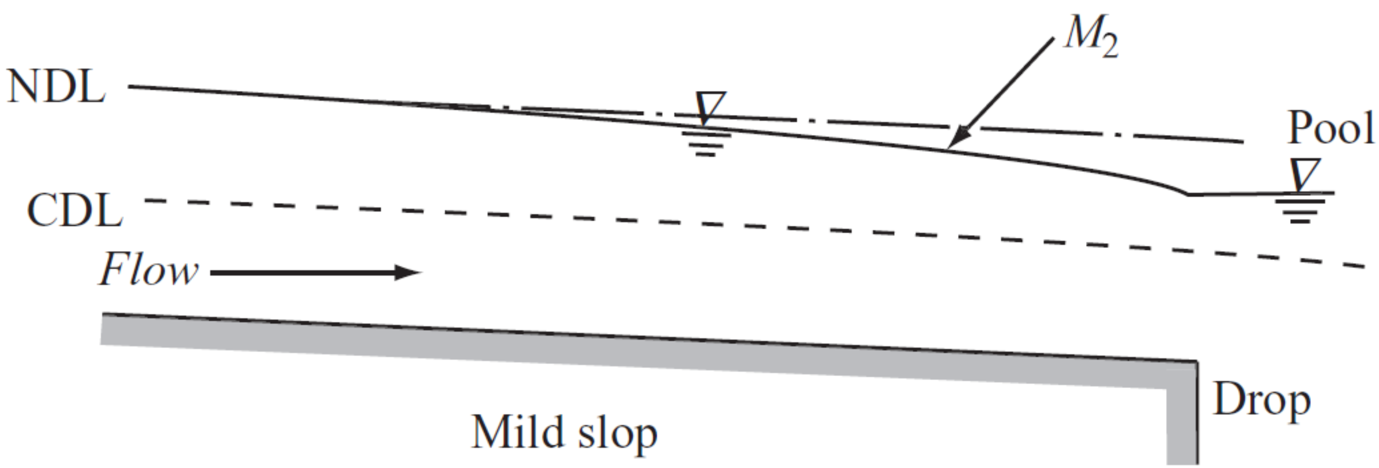


Fig. 4.4 (b) *M_2 profile*

WATER SURFACE PROFILES OF GVF

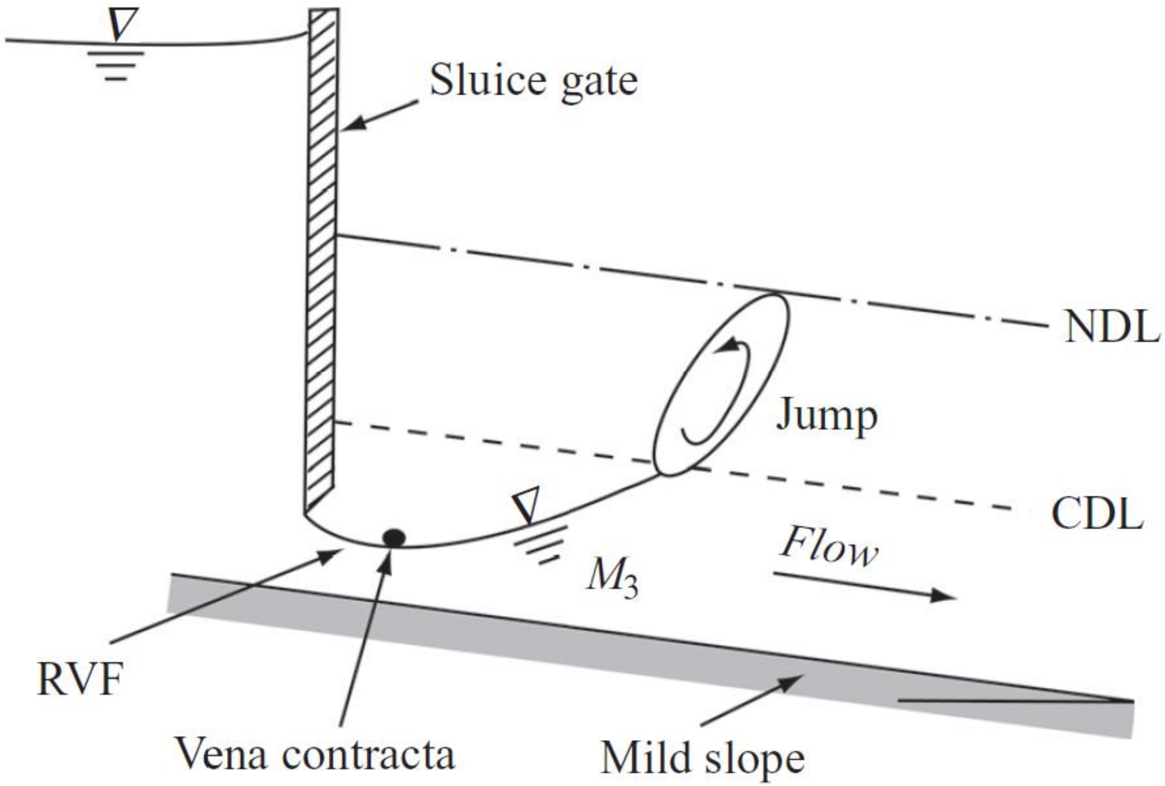


Fig. 4.4 (c) *M_3 profile*

WATER SURFACE PROFILES OF GVF

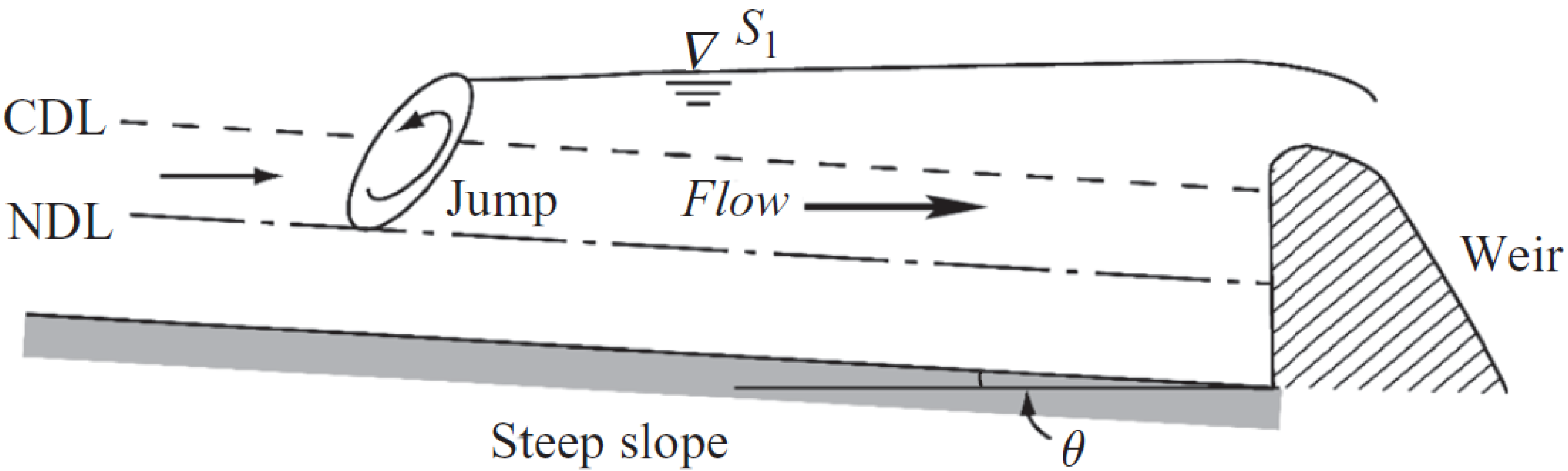


Fig. 4.4(d) S_1 *profile*

WATER SURFACE PROFILES OF GVF

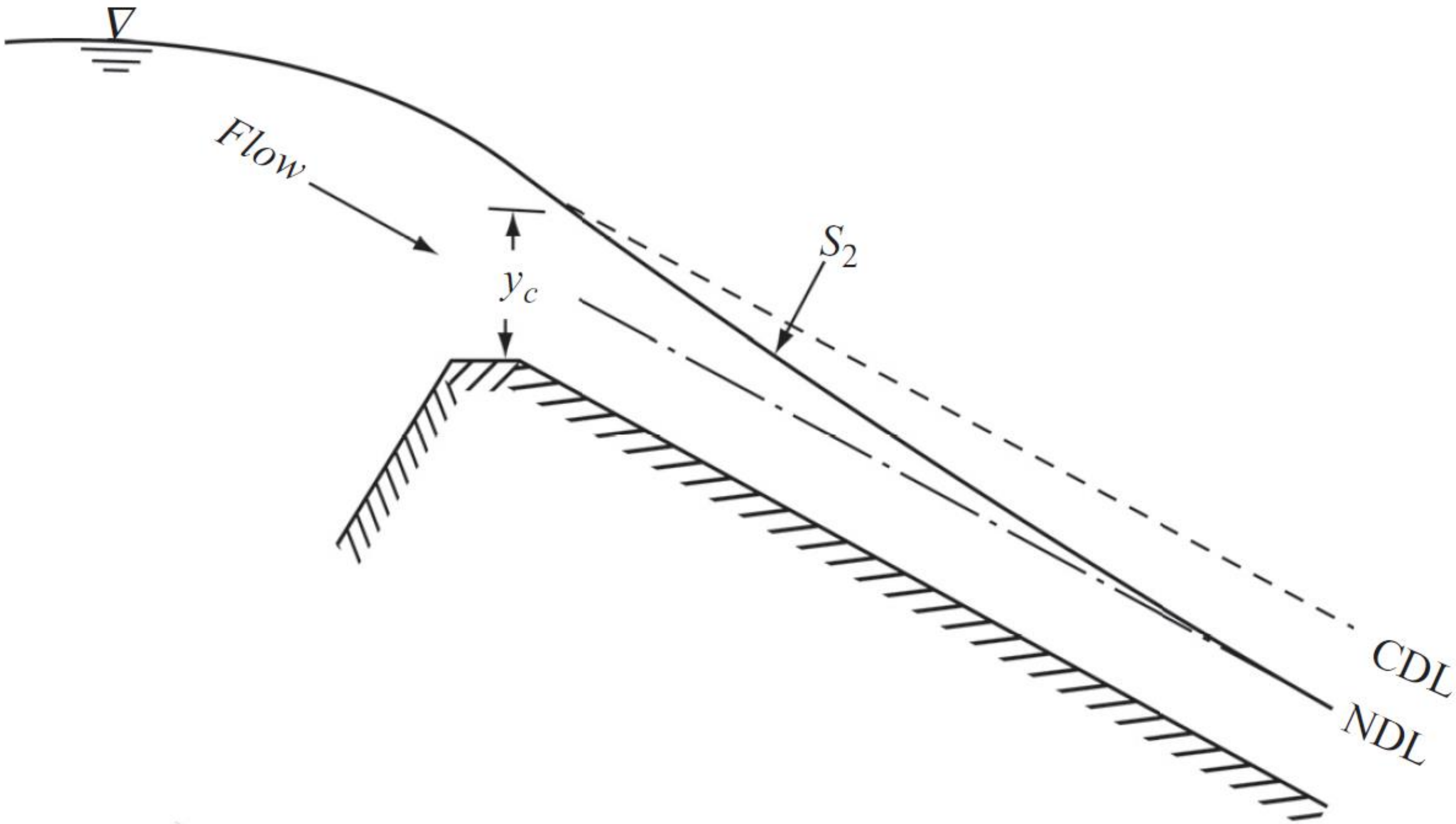


Fig. 4.4(e) *S_2 profile*

WATER SURFACE PROFILES OF GVF

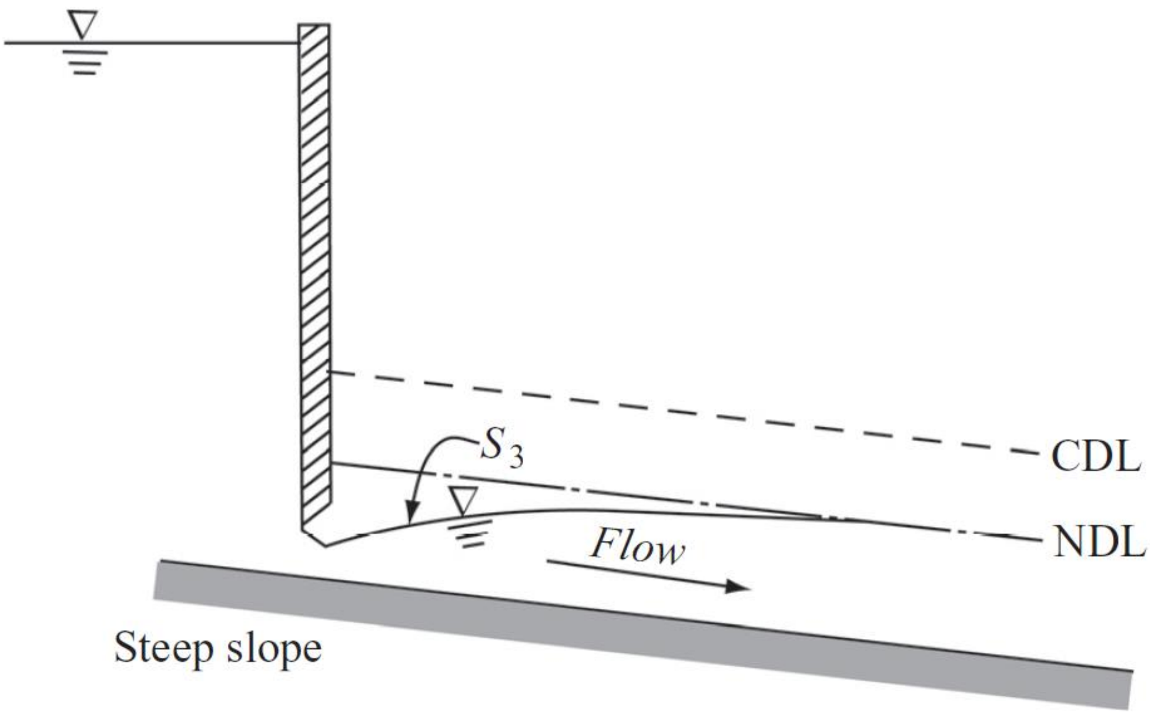


Fig. 4.4(f) *S_3 profile*

WATER SURFACE PROFILES OF GVF

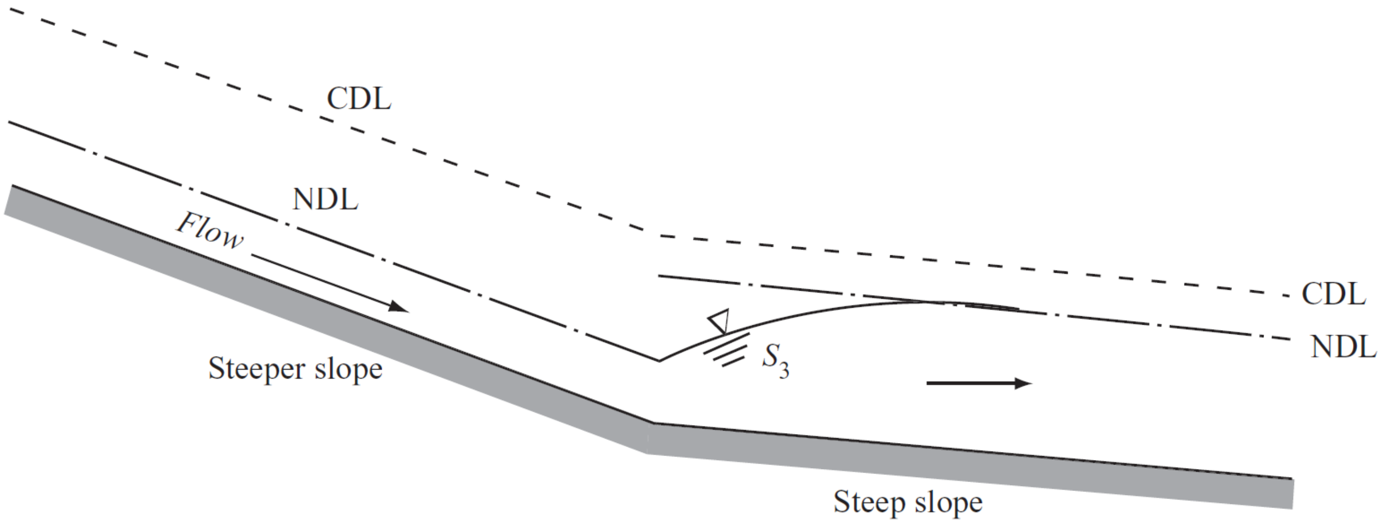


Fig. 4.4(g) *S_3 profile*

WATER SURFACE PROFILES OF GVF

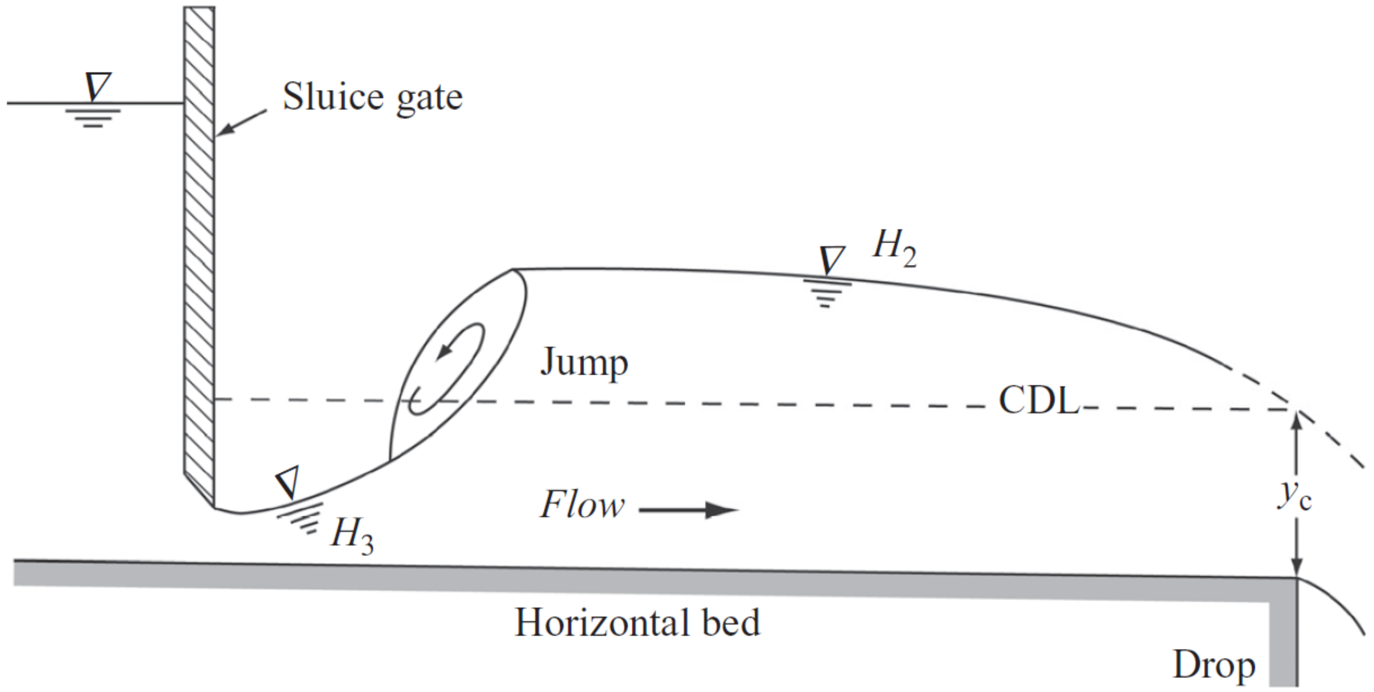


Fig. 4.4(h) *H_2 and H_3 profiles*

WATER SURFACE PROFILES OF GVF

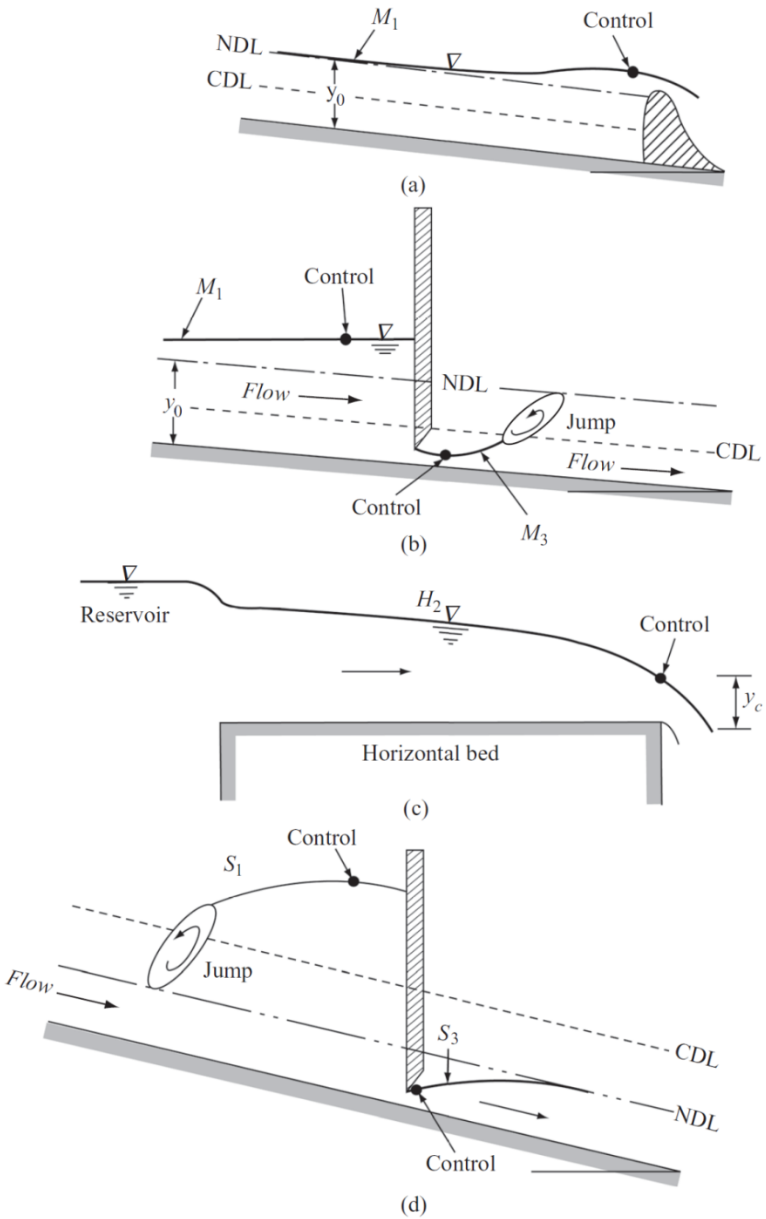
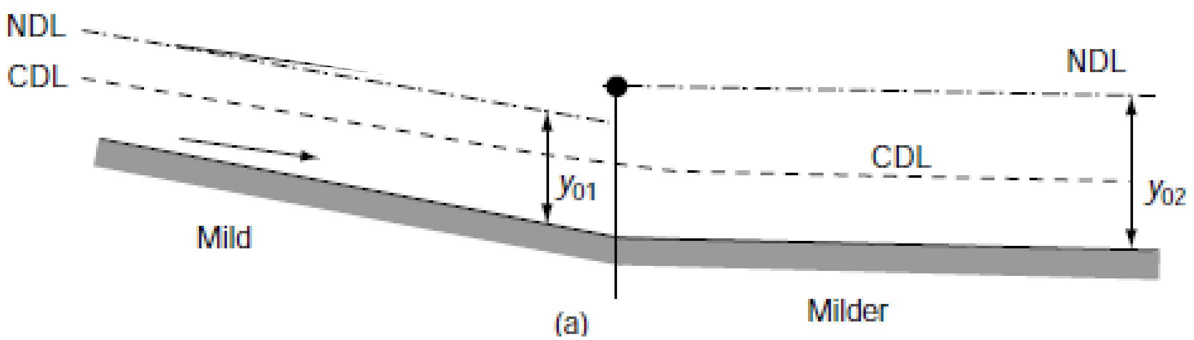


Fig. 4.5 *(contd)*

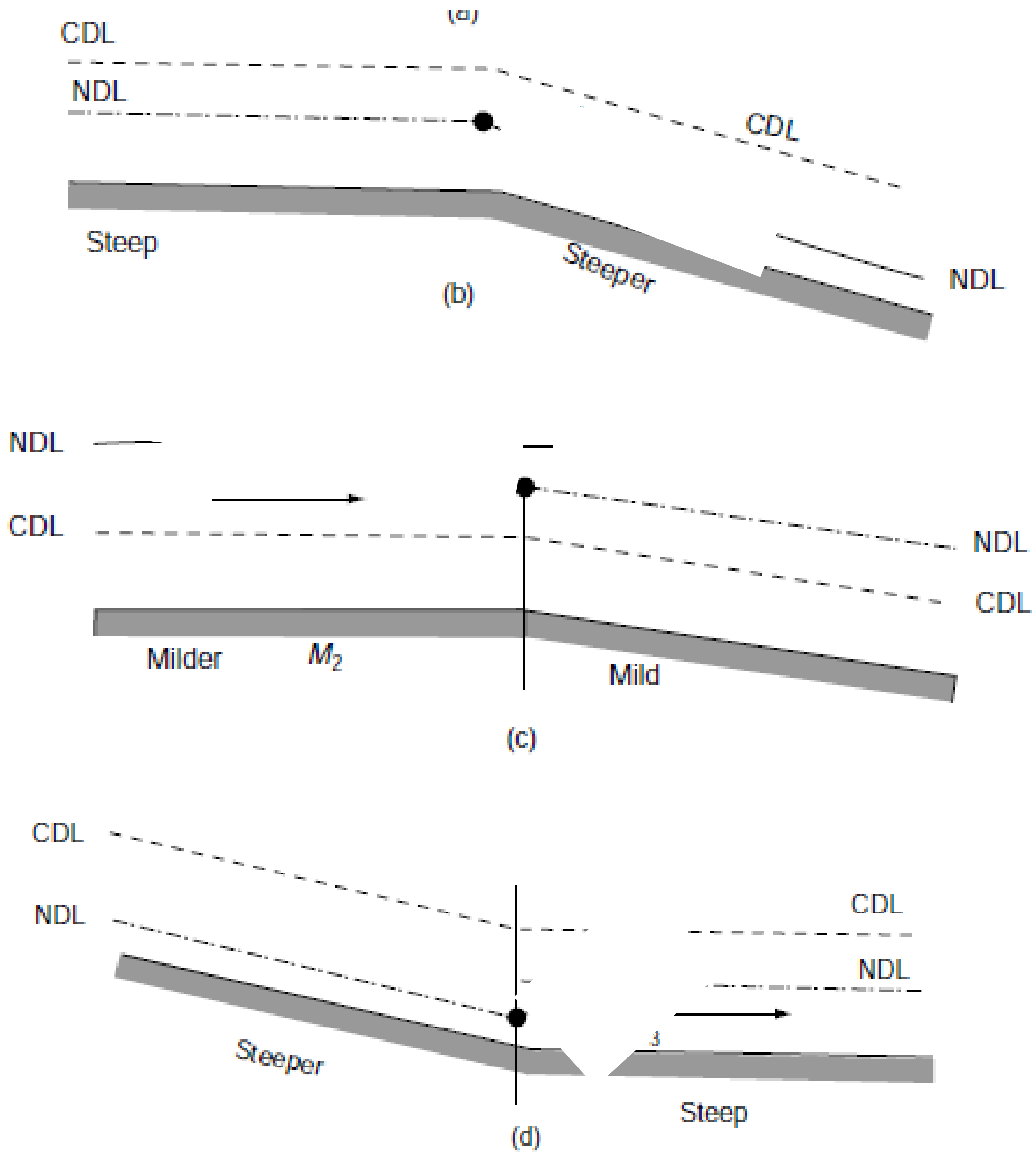
WATER SURFACE PROFILES OF GVF

When y is in region 2 $$\frac{dy}{dx} = -ve\ always$$

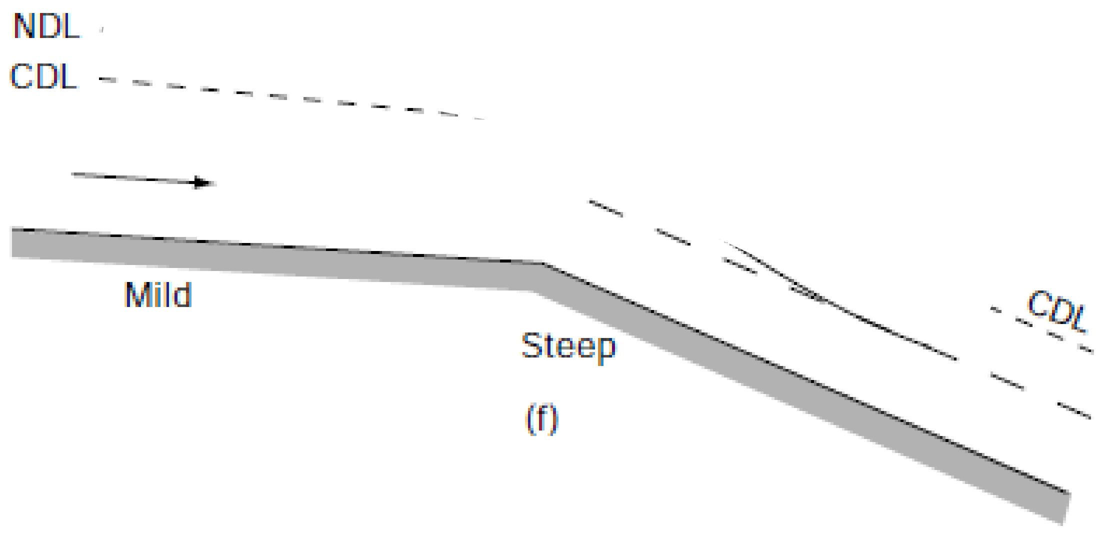
When y is in region 3 $$\frac{dy}{dx} = +ve\ always$$

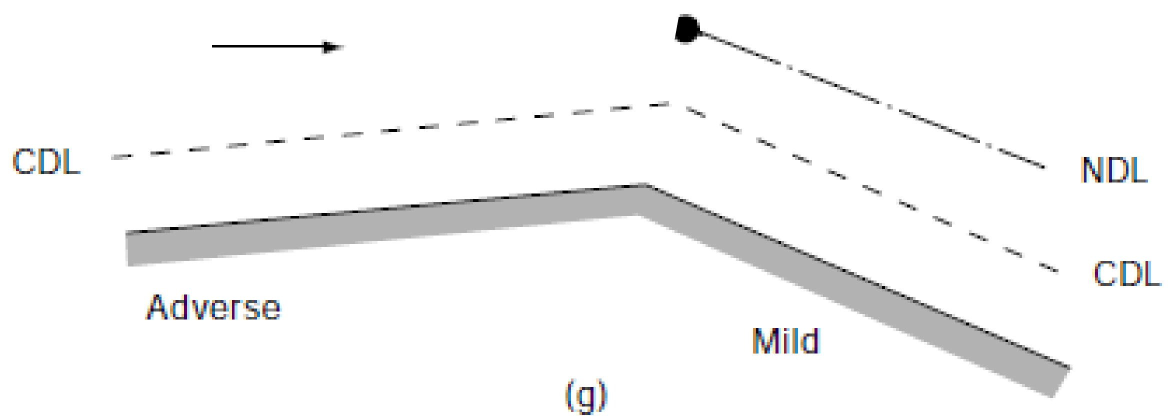


WATER SURFACE PROFILES OF GVF



WATER SURFACE PROFILES OF GVF





WATER SURFACE PROFILES OF GVF

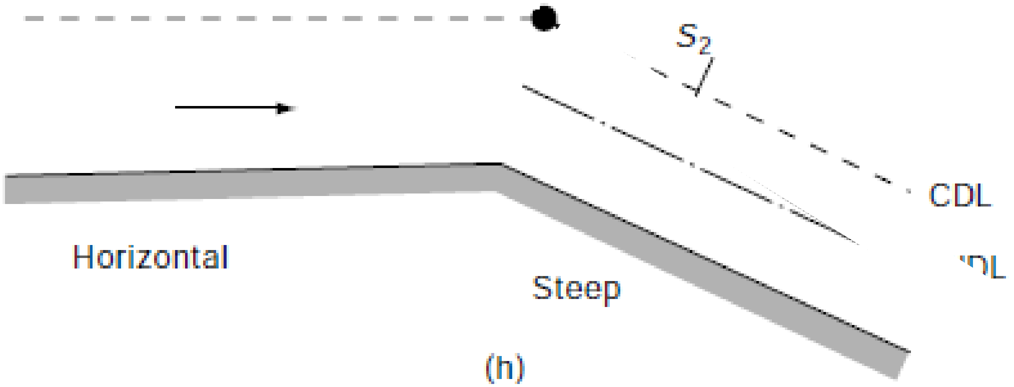
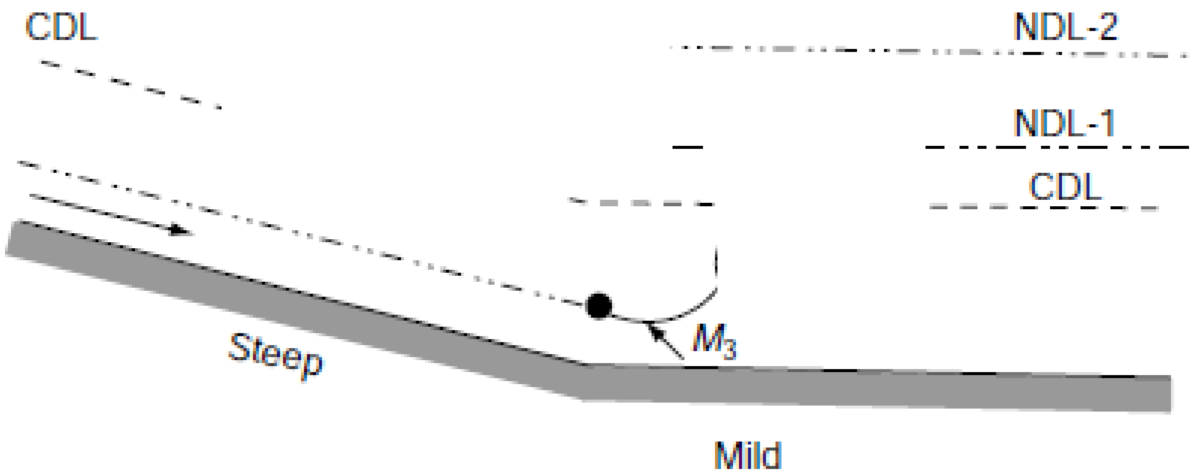


Fig. 4.6 *GVT profiles at break in grades*

WATER SURFACE PROFILES OF GVF



(e)

TODAY’S DEAL

ASSIGNMENT -4

SOLVE ANY 10 UNSOLVED QUESTIONS FROM THE CHAPTER : GRADUALLY VARIED FLOW

LAST DATE: TH NOVEBER 2020, SUNDAY

MAIL TO : hhmc2021@gmail.com

THE END